MINNESOTA DEPARTMENT OF TRANSPORTATION
STATE AID FOR LOCAL TRANSPORTATION DIVISION
Technical Memorandum No. 04-SA-03
April 30, 2004

TO: County Engineers
City Engineers
Consulting Engineers
District State Aid Engineers

FROM: Julie Skallman
State Aid Engineer

SUBJECT: Vertical Curve Charts

The vertical curve charts of the Road Design Manual (attached) have been modified to reflect the increased object height allowed by AASHTO's "A Policy on Geometric Design of Highways and Streets" (Green Book). These charts may be used for state aid and federal aid projects at this time. The State Aid Manual will be revised accordingly and distributed shortly.

The Road Design Manual is available on-line at http://www.dot.state.mn.us/tecsup/rdm/index.html.

Note that because the increased object height will result in sharper vertical curves, sight distance of cross traffic at intersecting roadways should be verified so that vehicles are able to see traffic approaching over crest vertical curves before deciding whether to enter the intersection.

If you have any questions concerning this memorandum, please contact Paul Stine, State Aid Operations Engineer, at (651) 296-9973.

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cc: DSAA

3-4.03 Vertical Curves

Mn/DOT's Technical Manual provides detailed explanations of the geometrics of vertical curves and the mathematics related to curve computations. Discussion in this section is limited to criteria and guides for design of vertical curves.

The principal concern in designing crest vertical curves is to ensure that at least the minimum stopping sight distance is provided. Headlight sight distance and rider comfort control the design of sag vertical curves. Two factors affect the availability of sight distance - the algebraic difference between gradients of the intersecting tangents, and the length of the vertical curve. With a small algebraic difference in grades, the length of the vertical curve may be relatively short. To obtain the same sight distance with a large algebraic difference in grades, a much longer vertical curve must be used.

The aesthetic minimum length for an algebraic difference (A) in grades of 1 percent or more is 1,000 ft. For less than 1%, the aesthetic minimum length should be "A" times 1,000 ft, but not to be less than 400 ft.

Conditions may make it necessary to go below the aesthetic minimum. The absolute minimum of vertical curve is 3 times the design speed ($L_{min} = 3V$). The vertical curve most often used in road design is the symmetric parabolic curve, where the distance from the Vertical Point of Curvature (VPC) to Vertical Point of Intersection (VPI) is equal to the distance from the VPI to the Vertical Point of Tangent (VPT). All equations for vertical curve design lengths are based on the symmetric parabolic curve. Unsymmetric parabolic vertical curves may be used for special situations, and the equations for design lengths will have to be derived.

K values will be used quite frequently when determining minimum design lengths of vertical curves for various design speeds. The K value is the horizontal distance in feet required to effect a 1% change in gradient, which is a measure of curvature. The K value in simple terms is L/A, and is useful in determining the horizontal distance from the VPC to the apex of a crest vertical curve or the low point in a sag vertical curve.

The equations used for vertical curves are as follows:

CREST VERTICAL CURVES

When S is less than or equal to L use,

$$L = \frac{AS^2}{100(\sqrt{2h_1} + \sqrt{2h_2})^2}$$

When S is greater than L use,

$$L = 2S - \frac{200(\sqrt{h_1} + \sqrt{h_2})^2}{A}$$

HEADLIGHT SAG VERTICAL CURVES

When S is less than or equal to L use,

$$L = \frac{AS^2}{400 + 3.5S}$$

When S is greater than L use,

$$L = 2S - \frac{400 + 3.5S}{A}$$

COMFORT SAG VERTICAL CURVES

$$L = \frac{AV^2}{46.5}$$

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For all conditions:

L	=	Minimum length of vertical curve, ft
A	=	Algebraic difference in grades, %
S	=	Sight distance, ft
V	=	Design speed, mph, for "S"
K	=	Rate of vertical curvature per change in grade given as feet per percent grade change
h_1	=	Height of eye of driver from pavement
h_2	=	Height of object from pavement

3-4.04 Sight Distance

Stopping, non-striping, passing, and decision sight distance values and their derivations are presented in Chapter 2. Two other useful sight distance controls are headlight sight distance and rider comfort sag used in the design of sag vertical curves. For headlight sight distance, the goal is to design the curve so that a vehicle's headlight will illuminate a minimum distance of road ahead equal to the stopping sight distance. The headlight is considered to be 2.0 ft high and have an upward divergence of 1 deg from the longitudinal axis of the vehicle. When full roadway lighting is available and anticipated to be available in the future, providing headlight sight distance may not be necessary. In that case, the comfort effect of change in vertical direction in a sag vertical curve, because of the combined gravitational and centrifugal forces, becomes the design control. From limited data, the consensus is that riding is comfortable on sag vertical curves when the centripetal acceleration does not exceed 1 ft/sec^2. The formula for this criteria was shown earlier.

The following will apply to the designing of vertical curves.

1. On undivided and divided multi-lane highways the stopping sight distances may be used as the minimum design for crest vertical curves. The graph in Figure 3-4.04A presents the design distance subject to the acceptable minimum values discussed previously. The figure is based on the desirable stopping sight distances rather than the minimum values.
2. Non-striping sight distance is considered the desirable design for 2-lane highways; stopping sight distance is considered the minimum design. Non-striping sight distances for vertical curves are presented in Figure 3-4.04B.
3. If possible, passing sight distance should be provided on 2-lane highways. Figure 3-4.04C yields the necessary lengths of crest vertical curves. As with horizontal curves, however, drivers may be reluctant to pass on crests even when adequate sight distance is available. Therefore, it is not warranted to provide passing sight distances at great additional costs, unless it is necessary to meet the frequency of passing opportunities discussed under the design standards for rural 2-lane highways in Chapter Two. Construction of passing lane sections may be considered as an alternate solution.
4. On sag vertical curves for all highways, the headlight sight distance values in Figure 3-4.04D will be the minimum lengths except where longer curves are needed to meet the minimum length of vertical curves as previously discussed. However, on fully lighted highways the comfort sag criteria can be used. ($L = AV^2/46.5$). See Figure 3-4.04E.
5. Whenever vertical curvature appears in combination with roadway elements that may complicate the highway information presented to the driver, an appropriate decision sight distance should be provided. Figure 3-4.04F provides the necessary information for 10 seconds of decision sight distance with variable heights of object. Depending on the individual conditions and the nature of the hazard, the designer must select 10 seconds or more of decision time and the appropriate object height. Chapter Two contains a detailed explanation of decision sight distance.

Figure 3-4.04G illustrates K-value versus design speed for all sight distance controls on vertical curves. The figure demonstrates the relative differences among the several criteria.

The preceding discussion presents minimum design values; if practical, greater distances should be provided. It should also be emphasized that the figure computations are predicated on tangent horizontal alignment; if a vertical and horizontal curve occurs together, the figures do not apply and an individual assessment is necessary.

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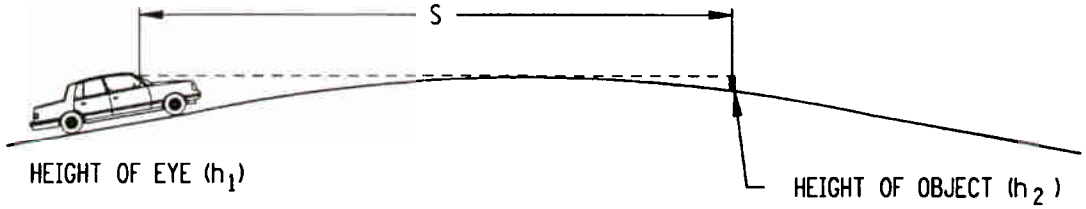


L = MINIMUM LENGTH OF VERTICAL CURVE, m
A = ALGEBRAIC DIFFERENCE IN GRADES, %
S = SIGHT DISTANCE, m
K = RATE OF VERTICAL CURVATURE PER CHANGE IN GRADE

USING CREST VERTICAL CURVE EQUATIONS WITH:
h_1 = 1.08 m h_2 = 0.60 m

WHEN S > L

$$L = 2S - \frac{658}{A}$$

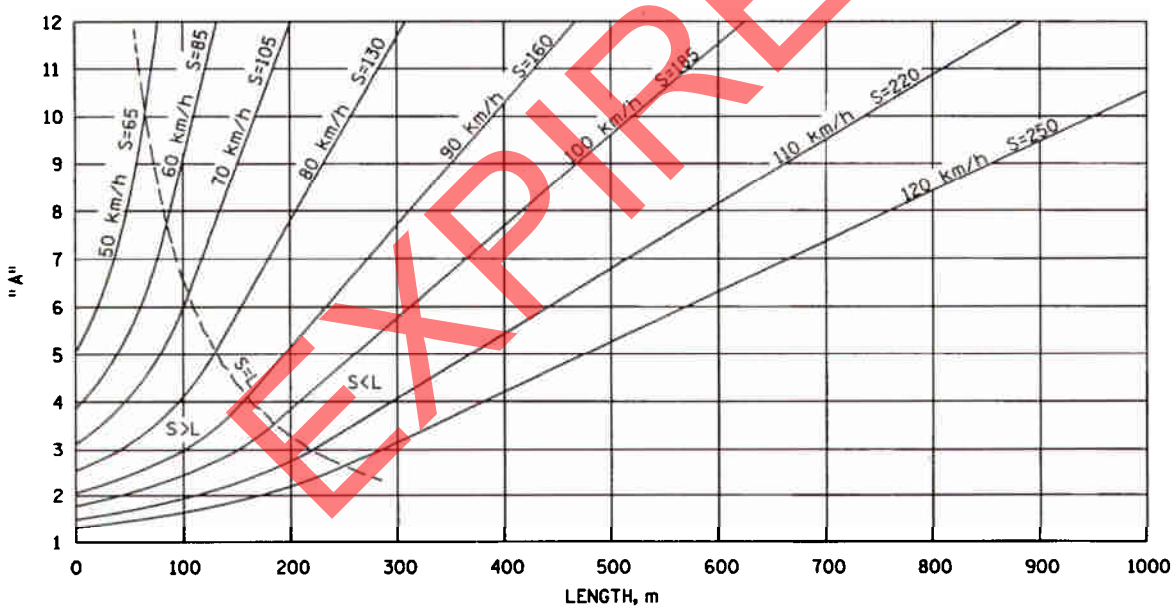
WHEN S ≤ L

$$L = \frac{AS^2}{658} = KA$$

DESIGN CONTROLS FOR CREST VERTICAL CURVES

K S > L	K * S ≤ L	DESIGN SPEED km/h	SIGHT DISTANCE m
USE GRAPH BELOW	7	50	65
	11	60	85
	17	70	105
	26	80	130
	39	90	160
	52	100	185
	74	110	220
	95	120	250

* OR USE GRAPH BELOW



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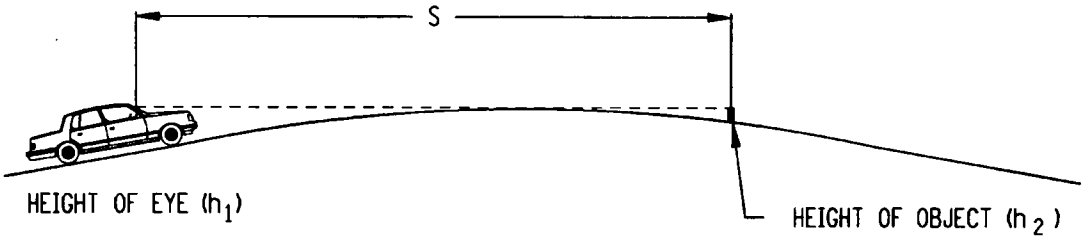
STOPPING SIGHT DISTANCE ON CREST VERTICAL CURVES
FIGURE 3-4.04A (METRIC)



L = MIN. LENGTH OF VERTICAL CURVE, FT.
A = ALGEBRAIC DIFFERENCE IN GRADES, %
S = SIGHT DISTANCE, FT.
K = RATE OF VERTICAL CURVATURE PER CHANGE IN GRADE

USING CREST VERTICAL CURVE EQUATIONS WITH:
h_1 = 3.50' h_2 = 2.00'

WHEN S > L

$$L = 2S - \frac{2158}{A}$$

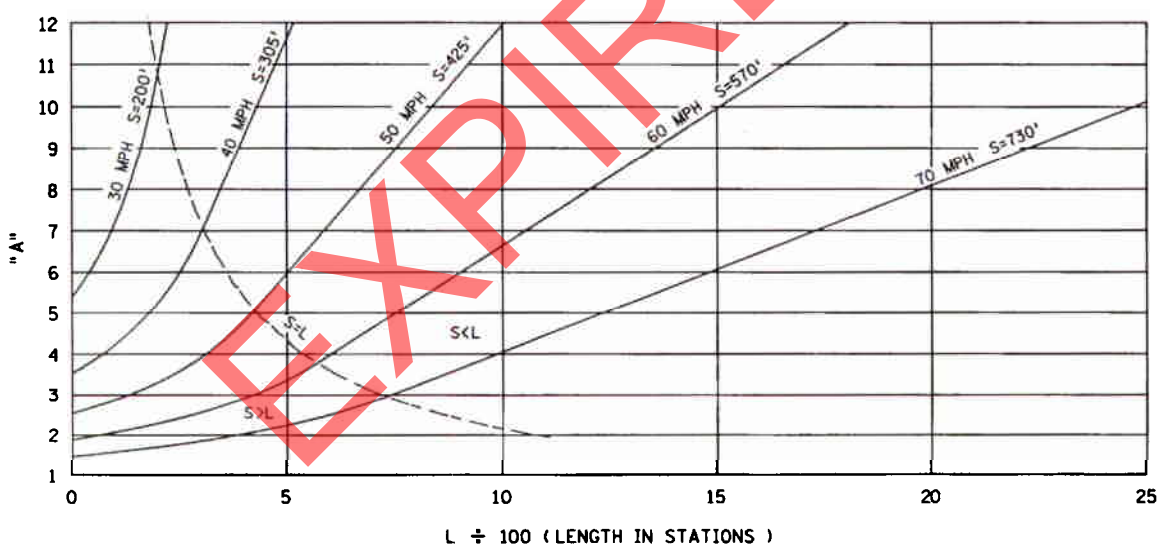
WHEN S ≤ L

$$L = \frac{AS^2}{2158} = KA$$

DESIGN CONTROLS FOR CREST VERTICAL CURVES

K S > L	K * S ≤ L	DESIGN SPEED (m.p.h.)	SIGHT DISTANCE (ft.)
USE GRAPH BELOW	19	30	200
	44	40	305
	84	50	425
	151	60	570
	247	70	730

* OR USE GRAPH BELOW



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STOPPING SIGHT DISTANCE ON CREST VERTICAL CURVES
FIGURE 3-4.04A (ENGLISH)

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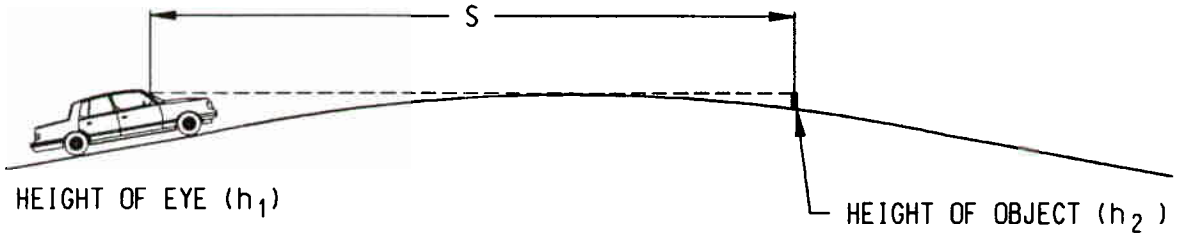


L = MIN. LENGTH OF VERTICAL CURVE, FT.
A = ALGEBRAIC DIFFERENCE IN GRADES, %
S = SIGHT DISTANCE, FT.
K = RATE OF VERTICAL CURVATURE PER CHANGE IN GRADE

USING CREST VERTICAL CURVE EQUATIONS WITH:
h_1 = 3.50' h_2 = 3.50'

WHEN S > L

$$L = 2S - \frac{2800}{A}$$

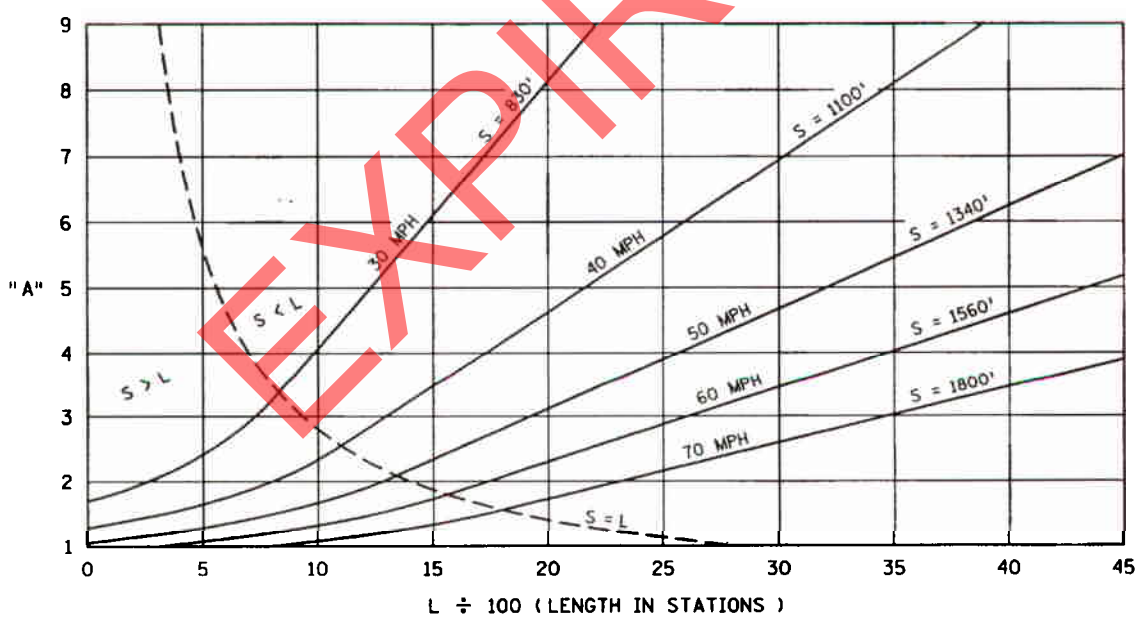
WHEN S ≤ L

$$L = \frac{AS^2}{2800} = KA$$

DESIGN CONTROLS FOR CREST VERTICAL CURVES

K S > L	K * S ≤ L	DESIGN SPEED (m.p.h.)	SIGHT DISTANCE (ft.)	DEGREE OF CURVE (min.)
USE GRAPH BELOW	246	30	830	2°20'
	432	40	1100	1°20'
	641	50	1340	0°54'
	869	60	1560	0°40'
	1157	70	1800	0°30'

* OR USE GRAPH BELOW



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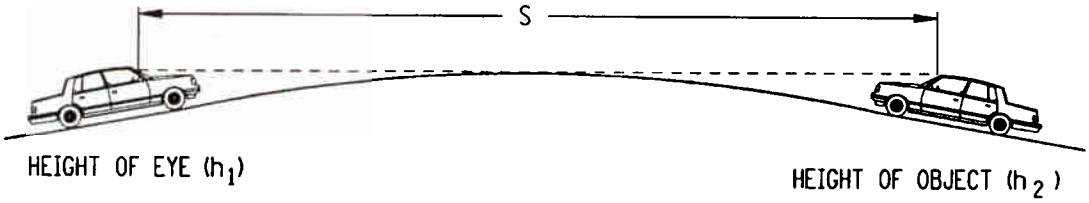
NON-STRIPING SIGHT DISTANCE ON CREST VERTICAL CURVES
Figure 3-4.04B



L = MINIMUM LENGTH OF VERTICAL CURVE, m
A = ALGEBRAIC DIFFERENCE IN GRADES, %
S = SIGHT DISTANCE, m
K = RATE OF VERTICAL CURVATURE PER CHANGE IN GRADE

USING CREST VERTICAL CURVE EQUATIONS WITH:
h_1 = 1.08 m h_2 = 1.08 m

WHEN S > L

$$L = 2S - \frac{864}{A}$$

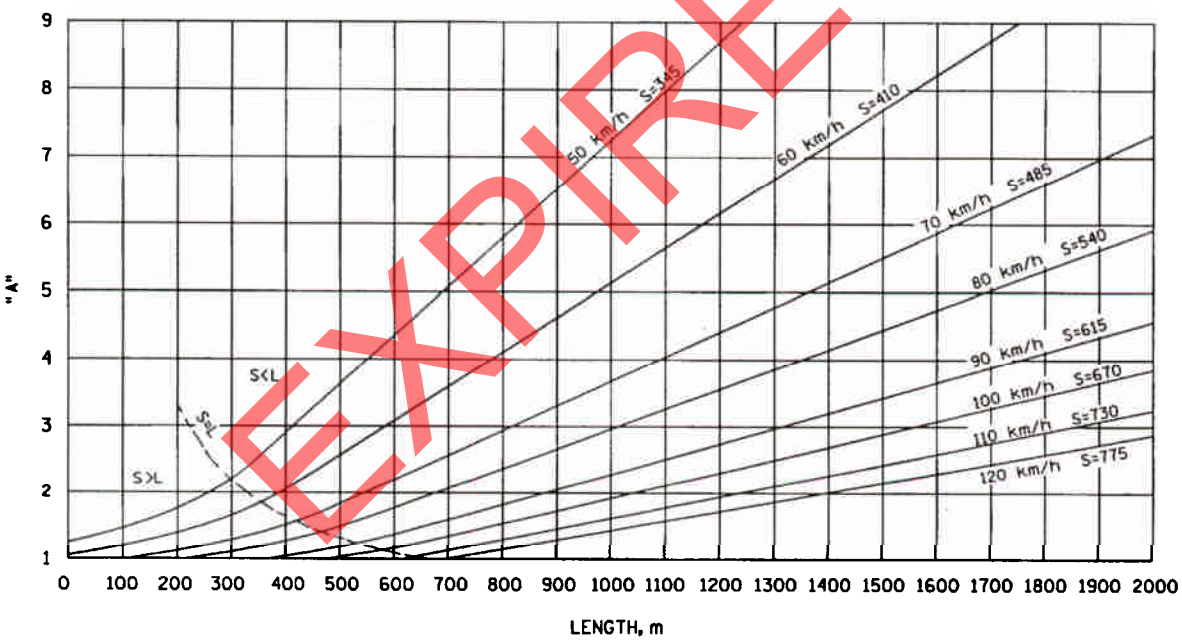
WHEN S ≤ L

$$L = \frac{AS^2}{864} = KA$$

DESIGN CONTROLS FOR CREST VERTICAL CURVES

K S > L	K * S ≤ L	DESIGN SPEED km/h	SIGHT DISTANCE m
USE GRAPH BELOW	138	50	345
	195	60	410
	272	70	485
	338	80	540
	438	90	615
	520	100	670
	617	110	730
	695	120	775

* OR USE GRAPH BELOW



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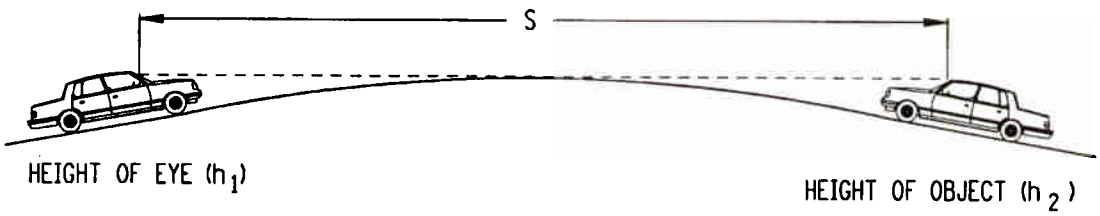
PASSING SIGHT DISTANCE ON CREST VERTICAL CURVES
FIGURE 3-4.04C (METRIC)



L = MIN. LENGTH OF VERTICAL CURVE, FT.
A = ALGEBRAIC DIFFERENCE IN GRADES, %
S = SIGHT DISTANCE, FT.
K = RATE OF VERTICAL CURVATURE PER CHANGE IN GRADE

USING CREST VERTICAL CURVE EQUATIONS WITH:
h_1 = 3.50' h_2 = 3.50'

WHEN S > L

$$L = 2S - \frac{2800}{A}$$

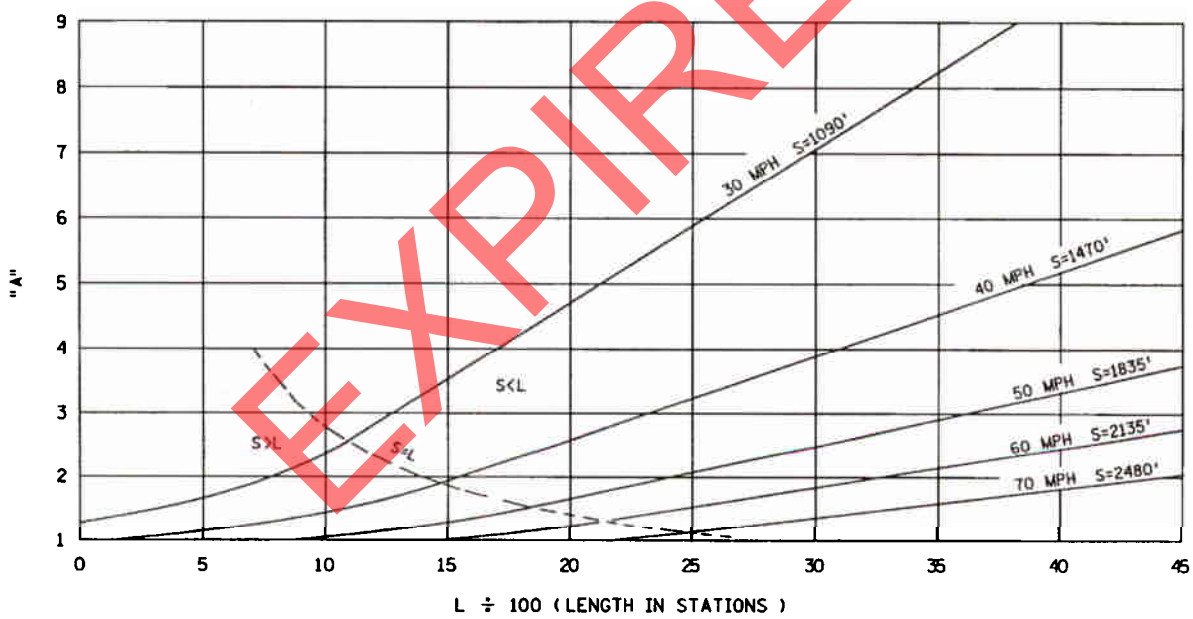
WHEN S ≤ L

$$L = \frac{AS^2}{2800} = KA$$

DESIGN CONTROLS FOR CREST VERTICAL CURVES

K S > L	K * S ≤ L	DESIGN SPEED (m.p.h.)	SIGHT DISTANCE (ft.)
USE GRAPH BELOW	424	30	1090
	772	40	1470
	1203	50	1835
	1628	60	2135
	2197	70	2480

* OR USE GRAPH BELOW



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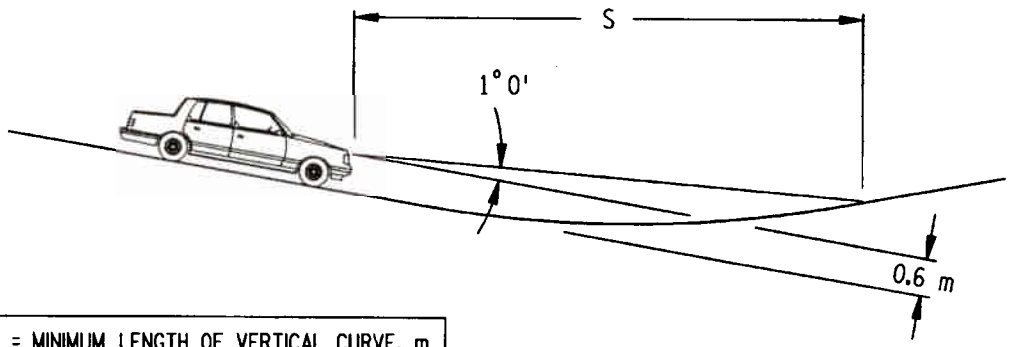
PASSING SIGHT DISTANCE ON CREST VERTICAL CURVES
FIGURE 3-4.04C (ENGLISH)



L = MINIMUM LENGTH OF VERTICAL CURVE, m
A = ALGEBRAIC DIFFERENCE IN GRADES, %
S = SIGHT DISTANCE, m
K = RATE OF VERTICAL CURVATURE PER CHANGE IN GRADE

USING HEADLIGHT SAG VERTICAL CURVE EQUATIONS:

WHEN S > L

$$L = 2S - \frac{120 + 3.5\,S}{A}$$

WHEN S ≤ L

$$L = \frac{AS^2}{120 + 3.5\,S} = KA$$

DESIGN CONTROLS FOR SAG VERTICAL CURVES

K S > L	K * S ≤ L	DESIGN SPEED km/h	SIGHT DISTANCE m
USE GRAPH BELOW	13	50	65
	18	60	85
	23	70	105
	30	80	130
	38	90	160
	45	100	185
	55	110	220
	63	120	250

* OR USE GRAPH BELOW

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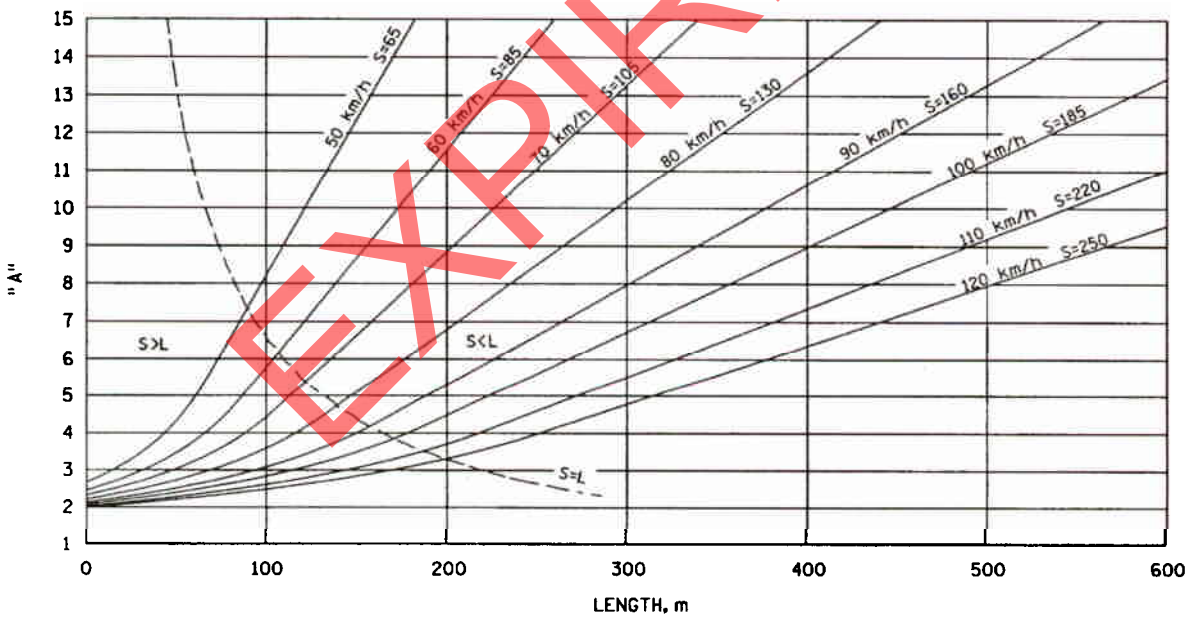
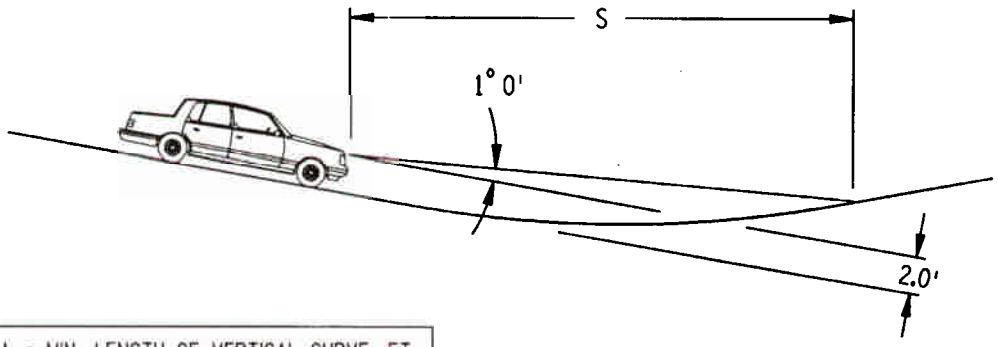


HEADLIGHT SIGHT DISTANCE ON SAG VERTICAL CURVES
FIGURE 3-4.04D (METRIC)



L = MIN. LENGTH OF VERTICAL CURVE, FT.
A = ALGEBRAIC DIFFERENCE IN GRADES, %
S = SIGHT DISTANCE, FT.
K = RATE OF VERTICAL CURVATURE PER CHANGE IN GRADE

USING HEADLIGHT SAG VERTICAL CURVE EQUATIONS:

WHEN S > L

$$L = 2S - \frac{400 + 3.5S}{A}$$

WHEN S ≤ L

$$L = \frac{AS^2}{400 + 3.5S} = KA$$

DESIGN CONTROLS FOR SAG VERTICAL CURVES

K S > L	K * S ≤ L	DESIGN SPEED (m.p.h.)	SIGHT DISTANCE (feet)
USE GRAPH BELOW	37	30	200
	64	40	305
	96	50	425
	136	60	570
	181	70	730

*OR USE GRAPH BELOW

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HEADLIGHT SIGHT DISTANCE ON SAG VERTICAL CURVES
FIGURE 3-4.04D (ENGLISH)

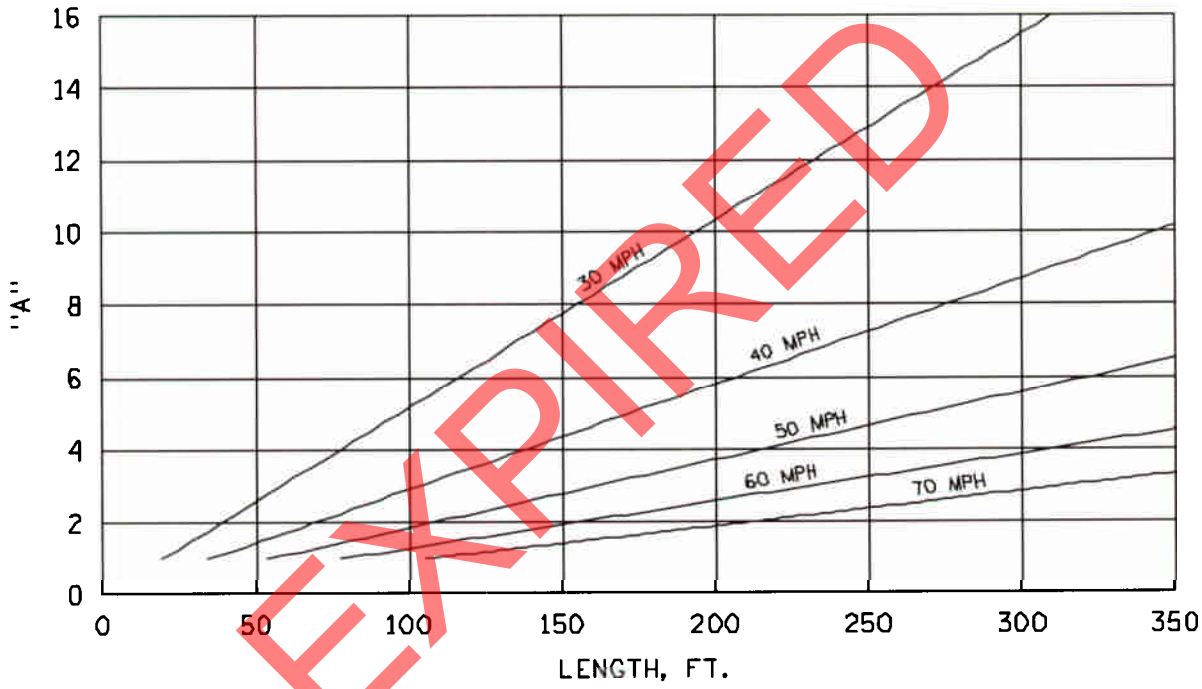
L = MIN. LENGTH OF VERTICAL CURVE, FT.
A = ALGEBRAIC DIFFERENCE IN GRADES, %
K = RATE OF VERTICAL CURVATURE PER CHANGE IN GRADE

USE COMFORT SAG VERTICAL CURVE EQUATION

$$L = \frac{AV^2}{46.5} = KA$$

DESIGN CONTROLS FOR COMFORT SAG VERTICAL CURVES

K	DESIGN SPEED (m.p.h.)	DEGREE OF CURVE (minimum)
19	30	30°09'
34	40	16°51'
54	50	10°37'
77	60	7°26'
105	70	5°27'



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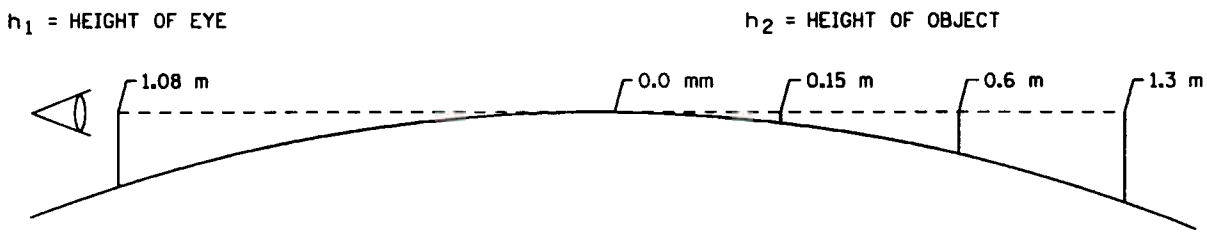
COMFORT SIGHT DISTANCE ON SAG VERTICAL CURVES
Figure 3-4.04E



CREST VERTICAL CURVE

DESIGN CONTROLS FOR CREST VERTICAL CURVES					
DESIGN SPEED km/h	SIGHT DISTANCE meters	K VALUES FOR EACH HEIGHT OF OBJECT (h_2)			
		h_2 = 0 m	h_2 = 0.15 m	h_2 = 0.6 m	h_2 = 1.3 m
50	139	90	48	30	20
60	167	130	69	43	29
70	194	176	93	58	40
80	222	230	122	75	52
90	250	292	155	95	66
100	278	361	191	118	82
110	306	438	232	143	99
120	333	518	274	169	117

SIGHT DISTANCE = 0.278 x t x V
t = DECISION TIME. USE 10 SEC.
(DIFFERENT VALUES MAY BE USED IF APPROPRIATE)
V = DESIGN SPEED, km/h

FOR GENERAL PURPOSES, AASHTO RECOMMENDS THE USE OF h_2 = 0.60 m
REFER TO CHAPTER 2, SECTION 2-5.09 FOR RECOMMENDATIONS ON VALUES OF h_2 FOR SPECIFIC LOCATIONS.

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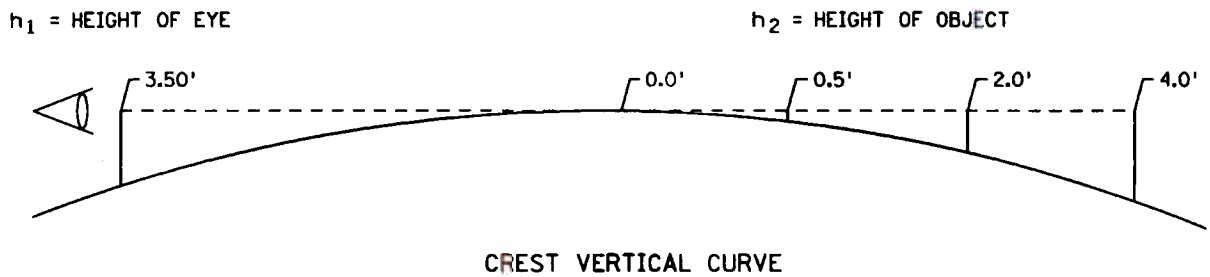
TEN SECOND DECISION SIGHT DISTANCE ON CREST VERTICAL CURVES
FIGURE 3-4.04F (METRIC)



CREST VERTICAL CURVE

DESIGN CONTROLS FOR CREST VERTICAL CURVES

DESIGN SPEED (m.p.h.)	SIGHT DISTANCE (feet)	K VALUES FOR EACH HEIGHT OF OBJECT (h_2)			
		h_2 = 0 FT.	h_2 = 0.5 FT.	h_2 = 2 FT.	h_2 = 4 FT.
30	440	277	146	90	65
40	587	492	259	160	115
50	733	768	404	249	179
60	880	1106	583	359	258
70	1027	1507	794	489	352

SIGHT DISTANCE = 1.47 x t x V
t = DECISION TIME. USE 10 SEC.
(DIFFERENT VALUES MAY BE USED IF APPROPRIATE)
V = DESIGN SPEED, m.p.h.

FOR GENERAL PURPOSES, AASHTO RECOMMENDS THE USE OF h_2 = 2.0 ft. REFER TO CHAPTER 2, SECTION 2-5.09 FOR RECOMMENDATIONS ON VALUES OF h_2 FOR SPECIFIC LOCATIONS.

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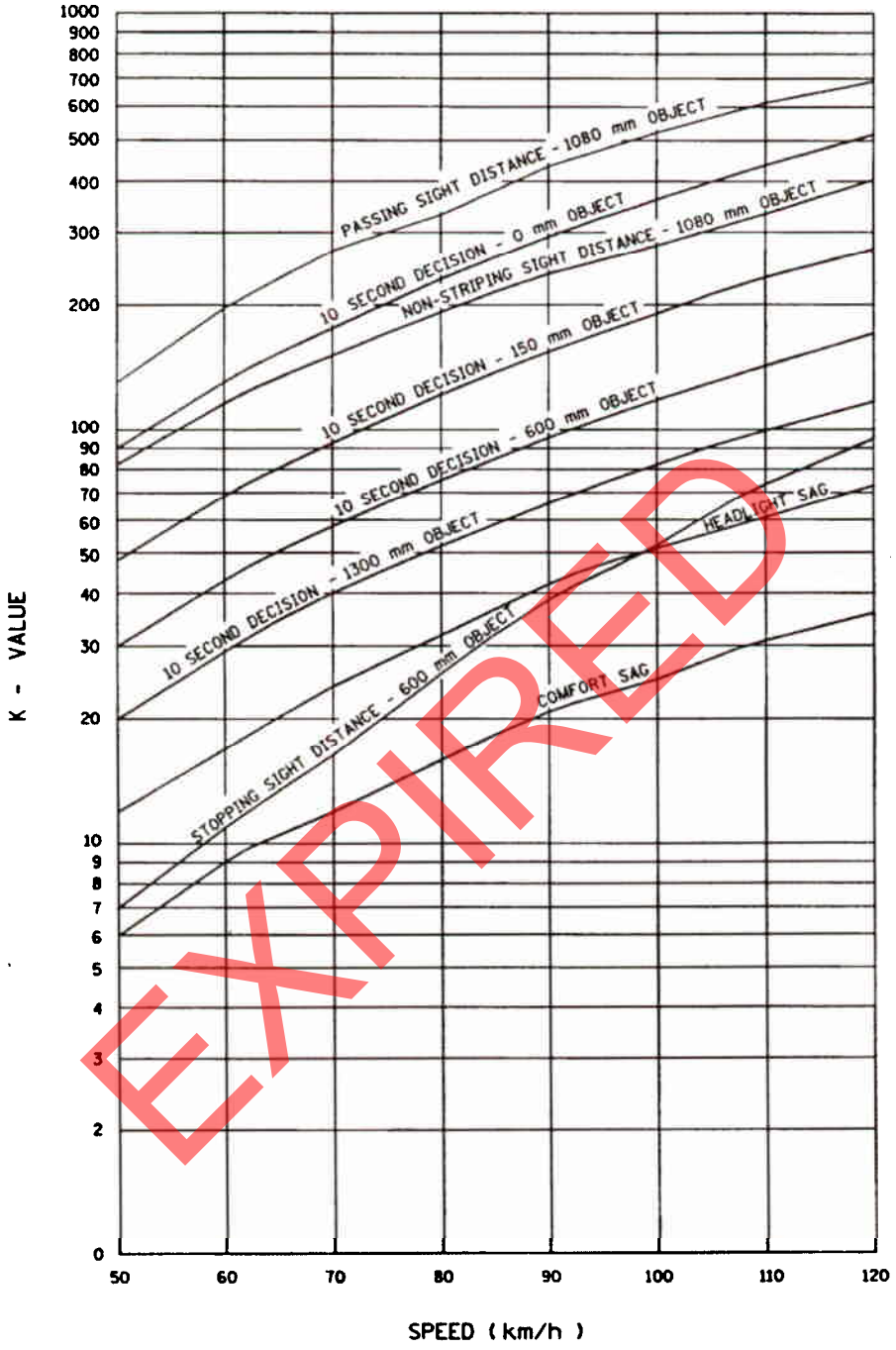
TEN SECOND DECISION SIGHT DISTANCE ON CREST VERTICAL CURVES
FIGURE 3-4.04F (ENGLISH)



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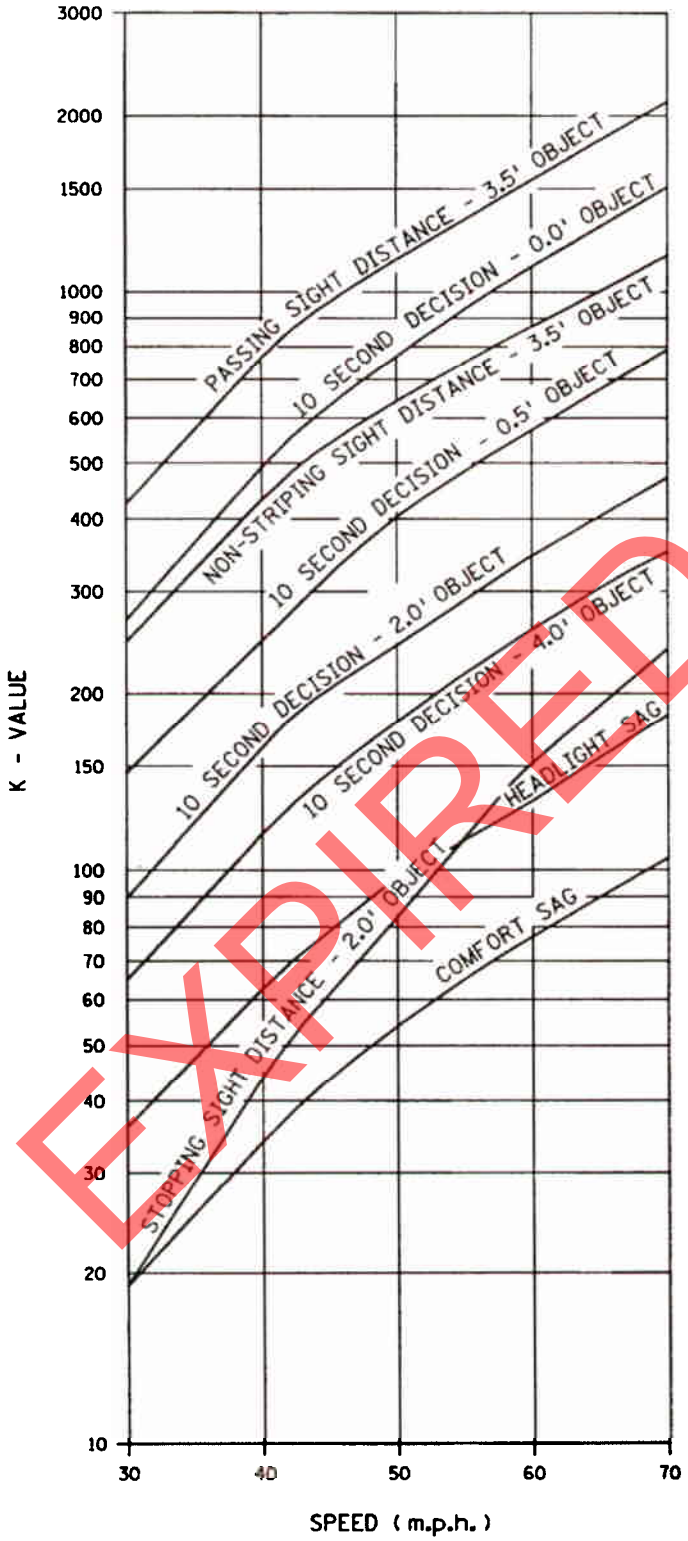
NOTE: THIS CHART IS FOR COMPARISON AND PRELIMINARY USE ONLY.
ACTUAL K VALUES SHOULD BE FOUND FROM THE APPROPRIATE PRECEDING CHARTS.

K VALUES ON VERTICAL CURVES
FIGURE 3-4.04G (METRIC)



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K VALUES ON VERTICAL CURVES
FIGURE 3-4.04G (ENGLISH)