MINNESOTA DEPARTMENT OF TRANSPORTATION
STATE AID FOR LOCAL TRANSPORTATION DIVISION
Technical Memorandum No. 04-SA-03
April 30, 2004

TO: County Engineers
    City Engineers
    Consulting Engineers
    District State Aid Engineers

FROM: Julie Skallman
      State Aid Engineer

SUBJECT: Vertical Curve Charts

The vertical curve charts of the Road Design Manual (attached) have been modified to reflect the increased object height allowed by AASHTO’s “A Policy on Geometric Design of Highways and Streets" (Green Book). These charts may be used for state aid and federal aid projects at this time. The State Aid Manual will be revised accordingly and distributed shortly.

The Road Design Manual is available on-line at http://www.dot.state.mn.us/tecsup/rdm/index.html.

Note that because the increased object height will result in sharper vertical curves, sight distance of cross traffic at intersecting roadways should be verified so that vehicles are able to see traffic approaching over crest vertical curves before deciding whether to enter the intersection.

If you have any questions concerning this memorandum, please contact Paul Stine, State Aid Operations Engineer, at (651) 296-9973.

cc: DSAA
3-4.03 **Vertical Curves**

Mn/DOT’s Technical Manual provides detailed explanations of the geometrics of vertical curves and the mathematics related to curve computations. Discussion in this section is limited to criteria and guides for design of vertical curves.

The principal concern in designing crest vertical curves is to ensure that at least the minimum stopping sight distance is provided. Headlight sight distance and rider comfort control the design of sag vertical curves. Two factors affect the availability of sight distance - the algebraic difference between gradients of the intersecting tangents, and the length of the vertical curve. With a small algebraic difference in grades, the length of the vertical curve may be relatively short. To obtain the same sight distance with a large algebraic difference in grades, a much longer vertical curve must be used.

The aesthetic minimum length for an algebraic difference (A) in grades of 1 percent or more is 1,000 ft. For less than 1%, the aesthetic minimum length should be "A" times 1,000 ft, but not to be less than 400 ft.

Conditions may make it necessary to go below the aesthetic minimum. The absolute minimum of vertical curve is 3 times the design speed ($L_{min} = 3V$). The vertical curve most often used in road design is the symmetric parabolic curve, where the distance from the Vertical Point of Curvature (VPC) to Vertical Point of Intersection (VPI) is equal to the distance from the VPI to the Vertical Point of Tangent (VPT). All equations for vertical curve design lengths are based on the symmetric parabolic curve. Unsymmetric parabolic vertical curves may be used for special situations, and the equations for design lengths will have to be derived.

K values will be used quite frequently when determining minimum design lengths of vertical curves for various design speeds. The K value is the horizontal distance in feet required to effect a 1% change in gradient, which is a measure of curvature. The K value in simple terms is $L/A$, and is useful in determining the horizontal distance from the VPC to the apex of a crest vertical curve or the low point in a sag vertical curve.

The equations used for vertical curves are as follows:

**CREST VERTICAL CURVES**

When $S$ is less than or equal to $L$ use,

$$L = \frac{AS^2}{100(\sqrt{2h_1} + \sqrt{2h_2})^2}$$

When $S$ is greater than $L$ use,

$$L = 2S - \frac{200(\sqrt{h_1} + \sqrt{h_2})^2}{A}$$

**HEADLIGHT SAG VERTICAL CURVES**

When $S$ is less than or equal to $L$ use,

$$L = \frac{AS^2}{400 + 3.5S}$$

When $S$ is greater than $L$ use,

$$L = 2S - \frac{400 + 3.5S}{A}$$

**COMFORT SAG VERTICAL CURVES**

$$L = \frac{AV^2}{46.5}$$
For all conditions:

\[ \begin{align*}
L &= \text{Minimum length of vertical curve, ft} \\
A &= \text{Algebraic difference in grades, \%} \\
S &= \text{ Sight distance, ft} \\
V &= \text{ Design speed, mph, for "S"} \\
K &= \text{ Rate of vertical curvature per change in grade given as feet per percent grade change} \\
h_1 &= \text{ Height of eye of driver from pavement} \\
h_2 &= \text{ Height of object from pavement}
\end{align*} \]

3-4.04 Sight Distance

Stopping, non-stripping, passing, and decision sight distance values and their derivations are presented in Chapter 2. Two other useful sight distance controls are headlight sight distance and rider comfort sag used in the design of sag vertical curves. For headlight sight distance, the goal is to design the curve so that a vehicle’s headlight will illuminate a minimum distance of road ahead equal to the stopping sight distance. The headlight is considered to be 2.0 ft high and have an upward divergence of 1 deg from the longitudinal axis of the vehicle. When full roadway lighting is available and anticipated to be available in the future, providing headlight sight distance may not be necessary. In that case, the comfort effect of change in vertical direction in a sag vertical curve, because of the combined gravitational and centrifugal forces, becomes the design control. From limited data, the consensus is that riding is comfortable on sag vertical curves when the centripetal acceleration does not exceed 1 ft/sec\(^2\). The formula for this criteria was shown earlier.

The following will apply to the designing of vertical curves.

1. On undivided and divided multi-lane highways the stopping sight distances may be used as the minimum design for crest vertical curves. The graph in Figure 3-4.04A presents the design distance subject to the acceptable minimum values discussed previously. The figure is based on the desirable stopping sight distances rather than the minimum values.

2. Non-stripping sight distance is considered the desirable design for 2-lane highways; stopping sight distance is considered the minimum design. Non-stripping sight distances for vertical curves are presented in Figure 3-4.04B.

3. If possible, passing sight distance should be provided on 2-lane highways. Figure 3-4.04C yields the necessary lengths of crest vertical curves. As with horizontal curves, however, drivers may be reluctant to pass on crests even when adequate sight distance is available. Therefore, it is not warranted to provide passing sight distances at great additional costs, unless it is necessary to meet the frequency of passing opportunities discussed under the design standards for rural 2-lane highways in Chapter Two. Construction of passing lane sections may be considered as an alternate solution.

4. On sag vertical curves for all highways, the headlight sight distance values in Figure 3-4.04D will be the minimum lengths except where longer curves are needed to meet the minimum length of vertical curves as previously discussed. However, on fully lighted highways the comfort sag criteria can be used. \((L = \sqrt{A/V^2/46.5})\). See Figure 3-4.04E.

5. Whenever vertical curvature appears in combination with roadway elements that may complicate the highway information presented to the driver, an appropriate decision sight distance should be provided. Figure 3-4.04F provides the necessary information for 10 seconds of decision sight distance with variable heights of object. Depending on the individual conditions and the nature of the hazard, the designer must select 10 seconds or more of decision time and the appropriate object height. Chapter Two contains a detailed explanation of decision sight distance.

Figure 3-4.04G illustrates \(K\)-value versus design speed for all sight distance controls on vertical curves. The figure demonstrates the relative differences among the several criteria.

The preceding discussion presents minimum design values; if practical, greater distances should be provided. It should also be emphasized that the figure computations are predicated on tangent horizontal alignment; if a vertical and horizontal curve occurs together, the figures do not apply and an individual assessment is necessary.
HEIGHT OF EYE \( (h_1) \)

HEIGHT OF OBJECT \( (h_2) \)

L = MINIMUM LENGTH OF VERTICAL CURVE, m
A = ALGEBRAIC DIFFERENCE IN GRADES, %
S = SIGHT DISTANCE, m
K = RATE OF VERTICAL CURVATURE
PER CHANGE IN GRADE

USING CREST VERTICAL CURVE EQUATIONS WITH:
\[ h_1 = 1.08 \text{ m} \quad h_2 = 0.60 \text{ m} \]

WHEN \( S > L \)
\[ L = 2S - \frac{658}{A} \]

WHEN \( S \leq L \)
\[ L = \frac{AS^2}{658} = KA \]

DESIGN CONTROLS FOR CREST VERTICAL CURVES

<table>
<thead>
<tr>
<th>K</th>
<th>( K )</th>
<th>DESIGN SPEED</th>
<th>SIGHT DISTANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>S &gt; L</td>
<td>S &lt; L</td>
<td>km/h</td>
<td>m</td>
</tr>
<tr>
<td>7</td>
<td>50</td>
<td>65</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>60</td>
<td>85</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>70</td>
<td>105</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>80</td>
<td>130</td>
<td></td>
</tr>
<tr>
<td>39</td>
<td>90</td>
<td>160</td>
<td></td>
</tr>
<tr>
<td>52</td>
<td>100</td>
<td>185</td>
<td></td>
</tr>
<tr>
<td>74</td>
<td>110</td>
<td>220</td>
<td></td>
</tr>
<tr>
<td>95</td>
<td>120</td>
<td>250</td>
<td></td>
</tr>
</tbody>
</table>

* OR USE GRAPH BELOW

STOPPING SIGHT DISTANCE ON CREST VERTICAL CURVES
FIGURE 3-4.04A (METRIC)
STopping sight distance on crest vertical curves
Figure 3-4.04a (English)
THIS PAGE INTENTIONALLY LEFT BLANK
HEIGHT OF EYE ($h_1$)

$S$

HEIGHT OF OBJECT ($h_2$)

$L$ = MIN. LENGTH OF VERTICAL CURVE, FT.
$A$ = ALGEBRAIC DIFFERENCE IN GRADES, %
$S$ = SIGHT DISTANCE, FT.
$K$ = RATE OF VERTICAL CURVATURE
PER CHANGE IN GRADE

**USING CREST VERTICAL CURVE EQUATIONS WITH:**

$h_1 = 3.50'$

$h_2 = 3.50'$

**WHEN $S > L$**

$L = \frac{2S - 2800}{A}$

**WHEN $S \leq L$**

$L = \frac{AS^2}{2800} = KA$

**DESIGN CONTROLS FOR CREST VERTICAL CURVES**

<table>
<thead>
<tr>
<th>$K$</th>
<th>$S &gt; L$</th>
<th>$S \leq L$</th>
<th>DESIGN SPEED (m.p.h.)</th>
<th>SIGHT DISTANCE (ft.)</th>
<th>DEGREE OF CURVE (min.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>USE</td>
<td>246</td>
<td>30</td>
<td>830</td>
<td>2°20'</td>
<td></td>
</tr>
<tr>
<td>GRAPH</td>
<td>432</td>
<td>40</td>
<td>1100</td>
<td>1°20'</td>
<td></td>
</tr>
<tr>
<td>BELOW</td>
<td>641</td>
<td>50</td>
<td>1340</td>
<td>0°54'</td>
<td></td>
</tr>
<tr>
<td></td>
<td>869</td>
<td>60</td>
<td>1560</td>
<td>0°40'</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1157</td>
<td>70</td>
<td>1800</td>
<td>0°30'</td>
<td></td>
</tr>
</tbody>
</table>

*OR USE GRAPH BELOW

**NON-STIPING SIGHT DISTANCE ON CREST VERTICAL CURVES**

Figure 3-4.04B
HEIGHT OF EYE ($h_1$) \hspace{10cm} HEIGHT OF OBJECT ($h_2$)

$L = \text{MINIMUM LENGTH OF VERTICAL CURVE, m}$

$A = \text{ALGEBRAIC DIFFERENCE IN GRADES, \%}$

$S = \text{SIGHT DISTANCE, m}$

$K = \text{RATE OF VERTICAL CURVATURE}$

$= \text{PER CHANGE IN GRADE}$

**USING CREST VERTICAL CURVE EQUATIONS WITH:**

$h_1 = 1.08 \text{ m}$ \hspace{1cm} $h_2 = 1.08 \text{ m}$

\[ L = \begin{cases} 2S - \frac{864}{A} & \text{WHEN } S > L \\ \frac{A S^2}{864} = \frac{K A}{L} & \text{WHEN } S \leq L \end{cases} \]

**DESIGN CONTROLS FOR CREST VERTICAL CURVES**

\[ \begin{array}{ccc} \text{K} & \text{K \times S \leq L} & \text{DESIGN SPEED} \\ \text{km/h} & \text{km/h} & \text{SIGHT DISTANCE} \\ \text{m} \end{array} \]

\[
\begin{array}{cccc}
138   & 50   & 345 \\
195   & 60   & 410 \\
272   & 70   & 485 \\
338   & 80   & 540 \\
438   & 90   & 615 \\
520   & 100  & 670 \\
617   & 110  & 730 \\
695   & 120  & 775 \\
\end{array}
\]

**USE GRAPH BELOW**

*OR USE GRAPH BELOW*

**PASSING SIGHT DISTANCE ON CREST VERTICAL CURVES**

**FIGURE 3-4.04C (METRIC)**
HEIGHT OF EYE \( h_1 \)

HEIGHT OF OBJECT \( h_2 \)

\[
L = \text{MIN. LENGTH OF VERTICAL CURVE, FT.}
\]

\[
A = \text{ALGEBRAIC DIFFERENCE IN GRADES, } \%.
\]

\[
S = \text{SIGHT DISTANCE, FT.}
\]

\[
K = \text{RATE OF VERTICAL CURVATURE}
\]

\[
\text{PER CHANGE IN GRADE}
\]

**USING CREST VERTICAL CURVE**

**EQUATIONS WITH:**

\[
h_1 = 3.50' \quad h_2 = 3.50'
\]

**WHEN** \( S > L \)

\[
L = 2S - \frac{2800}{A}
\]

**WHEN** \( S \leq L \)

\[
L = \frac{AS^2}{2800} = KA
\]

**DESIGN CONTROLS FOR CREST VERTICAL CURVES**

<table>
<thead>
<tr>
<th>( K )</th>
<th>( K \times L )</th>
<th>\text{DESIGN SPEED (m.p.h.)}</th>
<th>\text{SIGHT DISTANCE (ft.)}</th>
</tr>
</thead>
<tbody>
<tr>
<td>USE</td>
<td>424</td>
<td>30</td>
<td>1090</td>
</tr>
<tr>
<td>GRAPH</td>
<td>712</td>
<td>40</td>
<td>1470</td>
</tr>
<tr>
<td>BELOW</td>
<td>1203</td>
<td>50</td>
<td>1835</td>
</tr>
<tr>
<td></td>
<td>1628</td>
<td>60</td>
<td>2135</td>
</tr>
<tr>
<td></td>
<td>2197</td>
<td>70</td>
<td>2480</td>
</tr>
</tbody>
</table>

* OR USE GRAPH BELOW

**PASSING SIGHT DISTANCE ON CREST VERTICAL CURVES**

**FIGURE 3-4.04C (ENGLISH)**
**L** = Minimum length of vertical curve, m

**A** = Algebraic difference in grades, %

**S** = Sight distance, m

**K** = Rate of vertical curvature per change in grade

**Using headlight sag vertical curve equations:**

**When** \( S > L \)

\[
L = 2S - \frac{120 + 3.5S}{A}
\]

**When** \( S \leq L \)

\[
L = \frac{AS^2}{120 + 3.5S} = KA
\]

**Design controls for sag vertical curves**

<table>
<thead>
<tr>
<th>( K )</th>
<th>S &gt; L</th>
<th>S ≤ L</th>
<th>Design Speed</th>
<th>Sight Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>50</td>
<td>65</td>
<td>13</td>
<td>65</td>
</tr>
<tr>
<td>18</td>
<td>60</td>
<td>85</td>
<td>18</td>
<td>85</td>
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<tr>
<td>23</td>
<td>70</td>
<td>105</td>
<td>23</td>
<td>105</td>
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<tr>
<td>30</td>
<td>80</td>
<td>130</td>
<td>30</td>
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<td>38</td>
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<td>160</td>
<td>38</td>
<td>160</td>
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<tr>
<td>45</td>
<td>100</td>
<td>185</td>
<td>45</td>
<td>185</td>
</tr>
<tr>
<td>55</td>
<td>110</td>
<td>220</td>
<td>55</td>
<td>220</td>
</tr>
<tr>
<td>63</td>
<td>120</td>
<td>250</td>
<td>63</td>
<td>250</td>
</tr>
</tbody>
</table>

*Or use graph below*

**Headlight sight distance on sag vertical curves**

*Figure 3-4.04D (Metric)*
L = MIN. LENGTH OF VERTICAL CURVE, FT.
A = ALGEBRAIC DIFFERENCE IN GRADES, %
S = SIGHT DISTANCE, FT.
K = RATE OF VERTICAL CURVATURE PER CHANGE IN GRADE

USING HEADLIGHT SAG VERTICAL CURVE EQUATIONS:

WHEN S > L
L = 2S - \frac{400 + 3.5S}{A}

WHEN S ≤ L
L = \frac{AS^2}{400 + 3.5S} = KA

**DESIGN CONTROLS FOR SAG VERTICAL CURVES**

<table>
<thead>
<tr>
<th>K</th>
<th>S &gt; L</th>
<th>K * S L</th>
<th>DESIGN SPEED (m.p.h.)</th>
<th>SIGHT DISTANCE (feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>USE</td>
<td>37</td>
<td>30</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td></td>
<td>64</td>
<td>40</td>
<td>305</td>
<td></td>
</tr>
<tr>
<td>GRAPH</td>
<td>96</td>
<td>50</td>
<td>425</td>
<td></td>
</tr>
<tr>
<td>BELOW</td>
<td>136</td>
<td>60</td>
<td>570</td>
<td></td>
</tr>
<tr>
<td></td>
<td>181</td>
<td>70</td>
<td>730</td>
<td></td>
</tr>
</tbody>
</table>

* OR USE GRAPH BELOW

**HEADLIGHT SIGHT DISTANCE ON SAG VERTICAL CURVES**
**FIGURE 3-4.04D (ENGLISH)**
L = MIN. LENGTH OF VERTICAL CURVE, FT.
A = ALGEBRAIC DIFFERENCE IN GRADES, %
K = RATE OF VERTICAL CURVATURE
   PER CHANGE IN GRADE

USE COMFORT SAG VERTICAL CURVE EQUATION

\[ L = \frac{AV^2}{46.5} = KA \]

### DESIGN CONTROLS
FOR COMFORT SAG VERTICAL CURVES

<table>
<thead>
<tr>
<th>K</th>
<th>DESIGN SPEED (m.p.h.)</th>
<th>DEGREE OF CURVE (minimum)</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>30</td>
<td>30° 09'</td>
</tr>
<tr>
<td>34</td>
<td>40</td>
<td>16° 51'</td>
</tr>
<tr>
<td>54</td>
<td>50</td>
<td>10° 37'</td>
</tr>
<tr>
<td>77</td>
<td>60</td>
<td>7° 26'</td>
</tr>
<tr>
<td>105</td>
<td>70</td>
<td>5° 27'</td>
</tr>
</tbody>
</table>

---

**COMFORT SIGHT DISTANCE ON SAG VERTICAL CURVES**

Figure 3-4.04E
\[ h_1 = \text{HEIGHT OF EYE} \]

\[ h_2 = \text{HEIGHT OF OBJECT} \]

**Crest Vertical Curve**

<table>
<thead>
<tr>
<th>Design Speed (km/h)</th>
<th>Sight Distance (m)</th>
<th>( h_2 = 0 \text{ m} )</th>
<th>( h_2 = 0.15 \text{ m} )</th>
<th>( h_2 = 0.6 \text{ m} )</th>
<th>( h_2 = 1.3 \text{ m} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>139</td>
<td>90</td>
<td>48</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>60</td>
<td>167</td>
<td>130</td>
<td>69</td>
<td>43</td>
<td>29</td>
</tr>
<tr>
<td>70</td>
<td>194</td>
<td>176</td>
<td>93</td>
<td>58</td>
<td>40</td>
</tr>
<tr>
<td>80</td>
<td>222</td>
<td>230</td>
<td>122</td>
<td>75</td>
<td>52</td>
</tr>
<tr>
<td>90</td>
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<td>292</td>
<td>155</td>
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<tr>
<td>100</td>
<td>278</td>
<td>361</td>
<td>191</td>
<td>118</td>
<td>82</td>
</tr>
<tr>
<td>110</td>
<td>306</td>
<td>438</td>
<td>232</td>
<td>143</td>
<td>99</td>
</tr>
<tr>
<td>120</td>
<td>333</td>
<td>518</td>
<td>274</td>
<td>169</td>
<td>117</td>
</tr>
</tbody>
</table>

\[ \text{SIGHT DISTANCE} = 0.278 \times t \times V \]

\[ t = \text{DECISION TIME. USE 10 SEC.} \]

\[ (\text{DIFFERENT VALUES MAY BE USED IF APPROPRIATE}) \]

\[ V = \text{DESIGN SPEED, km/h} \]

For general purposes, AASHTO recommends the use of \( h_2 = 0.60 \text{ m} \).

Refer to Chapter 2, Section 2-5.09 for recommendations on values of \( h_2 \) for specific locations.

**TEN SECOND DECISION SIGHT DISTANCE ON CREST VERTICAL CURVES**

**FIGURE 3-4.04F (METRIC)**
\( h_1 = \text{HEIGHT OF EYE} \)

\( h_2 = \text{HEIGHT OF OBJECT} \)

**CREST VERTICAL CURVE**

**DESIGN CONTROLS FOR CREST VERTICAL CURVES**

<table>
<thead>
<tr>
<th>DESIGN SPEED (m.p.h.)</th>
<th>SIGHT DISTANCE (feet)</th>
<th>( K ) VALUES FOR EACH HEIGHT OF OBJECT (( h_2 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( h_2 = 0 \text{ FT.} )</td>
</tr>
<tr>
<td>30</td>
<td>440</td>
<td>277</td>
</tr>
<tr>
<td>40</td>
<td>587</td>
<td>492</td>
</tr>
<tr>
<td>50</td>
<td>733</td>
<td>768</td>
</tr>
<tr>
<td>60</td>
<td>880</td>
<td>1106</td>
</tr>
<tr>
<td>70</td>
<td>1027</td>
<td>1507</td>
</tr>
</tbody>
</table>

\[
\text{SIGHT DISTANCE} = 1.47 \times t \times V \\
\text{t = DECISION TIME. USE 10 SEC.} \\
\text{(DIFFERENT VALUES MAY BE USED IF APPROPRIATE)} \\
\text{V = DESIGN SPEED, m.p.h.}
\]

FOR GENERAL PURPOSES, AASHTO RECOMMENDS THE USE OF \( h_2 = 2.0 \text{ FT.} \). REFER TO CHAPTER 2, SECTION 2-5.09 FOR RECOMMENDATIONS ON VALUES OF \( h_2 \) FOR SPECIFIC LOCATIONS.

TEN SECOND DECISION SIGHT DISTANCE ON CREST VERTICAL CURVES

FIGURE 3-4.04F (ENGLISH)
K VALUES ON VERTICAL CURVES

FIGURE 3-4.04G (METRIC)

NOTE: THIS CHART IS FOR COMPARISON AND PRELIMINARY USE ONLY. ACTUAL K VALUES SHOULD BE FOUND FROM THE APPROPRIATE PRECEDING CHARTS.
NOTE: THIS CHART IS FOR COMPARISON AND PRELIMINARY USE ONLY. ACTUAL K VALUES SHOULD BE FOUND FROM THE APPROPRIATE PRECEDING CHARTS.

K VALUES ON VERTICAL CURVES
FIGURE 3-4.04G (ENGLISH)