

Project TPF 5(341): Permeability of Base Aggregate and Sand

INTERIM REPORT: TASK 2 – LITERATURE REVIEW

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Empirical and Theoretical Relations for Estimating Permeability of Coarse Aggregates and Sands

Methods based on Gradation

Numerous methods to estimate hydraulic conductivity from gradation (grain-size distribution) are summarized in Table 1 (Rosas et al. 2013). The table includes the primary reference, governing equation, applicable variables and corresponding units (where available), recommended values of constants, and a summary of use/constraints for each relation.

Table 1. Summary table of hydraulic conductivity estimation methods based upon gradation characteristics.

Method	Equation	Variable and Unit Definition	β	Use
Alyamani and Sen (1993)	$K \left[\frac{m}{d} \right] = \beta [I_0 + 0.025(d_{50} - d_{10})]^2$	I_0 = the intercept in mm of the line formed by d_{50} [mm] and d_{10} [mm] with the grain-size axis	1300	Well-graded sample
Barr (2000)	$K \left[\frac{m}{s} \right] = \beta \frac{\rho g}{\mu} n m^2$ $m = \frac{n}{s}$ $S = C_s S_0 (1 - n)$ $S_0 = \sum_i S_{oi}$ $S_{oi} = \frac{3}{r_i} w f_i$	m = the hydraulic radius S = the surface area C_s = a surface area adjusting parameter S_{oi} = the surface area per unit mass of solid material r = the radius of the sphere representing the grain (sieve size), in meters $w f_i$ = the weight fraction retained in sieve i ρ = the density of the fluid g = the gravitational constant of the fluid μ = the dynamic viscosity of the fluid	1/5	$1 < C_s < 1.35$
Beyer (1964)	$K \left[\frac{m}{s} \right] = \beta \frac{g}{v} \log \frac{500}{C} d_{10}^2$ $C = \frac{d_{60}}{d_{10}}$	C = the coefficient of uniformity g = the gravitational constant of the fluid v = Kinematic viscosity of the fluid	6×10^{-4}	$0.64 \text{ mm} < d_{10} < 0.6 \text{ mm}$ $1 < C < 20$

		$v = \mu / \rho$, where μ is the dynamic viscosity, and ρ is the density.		
Chapuis <i>et al.</i> (2005)	$K \left[\frac{\text{cm}}{\text{s}} \right] = \beta \left(\frac{d_{10}^2 e^3}{1 + e} \right)^{0.7825}$ $e = \frac{n}{1 - n}$	e = the void ratio d_{10} = in mm	2.4622	$0.03 \text{ mm} < d_{10} < 3 \text{ mm}$ $0.3 < e < 0.7$
Fair and Hatch (1933)	$K \left[\frac{\text{m}}{\text{s}} \right] = \beta \frac{\rho g}{\mu} \frac{n^3}{(1 - n)^2} \frac{1}{m \left(\frac{\theta}{100} \sum_i \frac{P_i}{d_{mi}} \right)}$ $P_i = 100 \cdot w f_i$ $d_{mi} = \sqrt{d_{s_i} \cdot d_{s_{i+1}}}$	m = a packing factor θ = a sand shape factor P = the percentage of sand held between adjacent sieves d_m = the geometric mean d_{s_i} = the size of the i sieve g = acceleration due to gravity μ = the dynamic viscosity of the fluid n = porosity	1	$m = 5$ $6 < \theta < 7.7$, spherical to angular respectively
Harleman <i>et al.</i> (1963)	$K \left[\frac{\text{m}}{\text{s}} \right] = \frac{\rho g}{\mu} d_{10}^2$		6.54×10^{-4}	
Hazen-original (1892)	$K \left[\frac{\text{m}}{\text{s}} \right] = \beta \frac{g}{v} [1 + 10(n - 0.26)] d_{10}^2$		6×10^{-4}	$0.1 \text{ mm} < d_{10} < 3 \text{ mm}$ $C < 5$
Hazen-new (modified)	$K \left[\frac{\text{m}}{\text{s}} \right] = \beta C d_{10}^2$	C = Hazen coefficient $1/[\text{cm} \cdot \text{s}]$ d_{10} = in cm	1	$100 < C < 150$
Kozeny (1953)	$K \left[\frac{\text{m}}{\text{s}} \right] = \beta \frac{g}{v} \frac{n^3}{(1 - n)^2} d_{10}^2$	g = the gravitational constant of the fluid v = Kinematic viscosity of the fluid	8.3×10^{-4}	Large-grain sands
Kozeny-Carman (Carman 1937, 1956; Kozeny 1927, 1953)	$K \left[\frac{\text{m}}{\text{s}} \right] = \beta \frac{\rho_w g}{\mu} \frac{n^3}{(1 - n)^2} d_{10}^2$		1/180	Silts, sands, and gravelly sands $d_{10} < 3 \text{ mm}$

Kruger (from Vukovic and Soro, 1992)	$K \left[\frac{m}{s} \right] = \beta \frac{g}{v} \frac{n}{(1-n)^2} d_e^2$ $\frac{1}{d_e} = \sum_{i=1}^n \frac{\Delta_{g_i}}{d_i}$	<p>g_i = the fractional percent weight retained on individual sieves</p> <p>d_i = the mean grain diameter in mm of the corresponding fraction</p>	4.3×10^{-5}	<p>$C > 5$</p> <p>Medium-grain sands</p>
Krumbein and Monk (1943)	$K [\text{darcy}] = \beta GM_{\xi}^2 e^{-1.31\sigma_{\phi}}$	<p>GM_{ξ} = the geometric mean diameter in mm</p> <p>σ_{ϕ} = the ϕ standard deviation</p> <p>K = the constant of proportionality in Darcy's original expression</p>	760	
NAVFAC DM7 (1974; from Chesnaux <i>et al.</i> 2011)	$K \left[\frac{m}{s} \right] = \beta 10^{1.291e - 0.6435 d_{10}^{0.5504 - 0.2937e}}$ $e = \frac{n}{1-n}$	<p>e = the void ratio</p> <p>d_{10} = in mm</p>	1	
Pavchich (Pravedny 1966)	$K \left[\frac{m}{s} \right] = \beta \frac{g}{v} d_{17}^2$		0.35	0.06mm < d_{17} < 1.5mm
Sauerbrei (from Vukovic and Soro 1992)	$K \left[\frac{m}{s} \right] = \beta \frac{g}{v} \frac{n^3}{(1-n)^2} d_{17}^2$		3.75×10^{-3}	<p>Sand and sandy clay</p> <p>$d_{17} < 0.5\text{mm}$</p>
Slichter (1899)	$K \left[\frac{m}{s} \right] = \beta \frac{g}{v} n^{3.287} d_{10}^2$		0.01	0.01mm < d_{10} < 5mm
Terzaghi (1925)	$K \left[\frac{m}{s} \right] = \beta \frac{g}{v} \left(\frac{n - 0.13}{\sqrt[3]{1-n}} \right)^2 d_{10}^2$		<p>10.7×10^{-3} for smooth grains</p> <p>6.1×10^{-3} for coarse grains</p>	Large-grain sands
U.S. Bureau of Reclamation (from Vukovic and Soro 1992)	$K \left[\frac{m}{s} \right] = \beta \frac{g}{v} d_{20}^{2.3}$	d_{20} = in mm	4.8×10^{-4}	<p>Medium-grain sands</p> <p>$C < 5$</p>
Zamarin (from Lu <i>et al.</i> 2012)	$K \left[\frac{m}{s} \right] = \beta_z \frac{g}{v} \frac{n^3}{(1-n)^2} d_e^2$	d_1 = the largest diameter of the finest fraction	8.2×10^{-3}	Large-grain sands

	$\frac{1}{d_e}$ $= \frac{3 \Delta_{g_1}}{2 d_1} + \sum_{i=2}^{i=n} \Delta_{g_i} \left(\frac{\ln \frac{d_i^g}{d_i^d}}{d_i^g - d_i^d} \right)$	<p>Δ_{g_1} = the weight of the material of the finest fraction in parts of the total weight</p> <p>d_i^g and d_i^d = maximum and minimum grain diameters of the fraction, respectively</p> <p>Δ_{g_i} = the fraction weight in parts of the total weight</p>		
<p>Zunker (1932; from Lu <i>et al.</i> 2012)</p>	$K \left[\frac{m}{s} \right] = \beta_z \frac{g}{v} \frac{n}{(1-n)^2} d_e^2$ $\frac{1}{d_e}$ $= \frac{3 \Delta_{g_1}}{2 d_1}$ $+ \sum_{i=2}^{i=n} \Delta_{g_i} \left(\frac{d_i^g - d_i^d}{d_i^g d_i^d \ln \frac{d_i^g}{d_i^d}} \right)$	<p>d_1 = the largest diameter of the finest fraction</p> <p>Δ_{g_1} = the weight of the material of the finest fraction in parts of the total weight</p> <p>d_i^g and d_i^d = maximum and minimum grain diameters of the fraction, respectively</p> <p>Δ_{g_i} = the fraction weight in parts of the total weight</p>	<p>2.4×10⁻³ for uniform sand with smooth, rounded grains</p> <p>1.4×10⁻³ for uniform composition with coarse grains</p> <p>1.2×10⁻³ for nonuniform composition</p> <p>0.7×10⁻³ for nonuniform compositions, clayey, with grains or irregular shape</p>	<p>Fine and medium-grain sands</p>

Methods based on Void Ratio or Porosity

Numerous methods to estimate hydraulic conductivity from void ratio (and often including grain-size distribution) are summarized in Table 2. The table includes the primary reference, governing equation, applicable variables and corresponding units, recommended values of constants, and a summary of use/constraints for each relation.

Table 2. Summary table of saturated hydraulic conductivity estimation methods based upon void ratio or porosity.

Reference	General Relationship	Equation	Notes
Dungca <i>et al.</i> (2018)	A multiple regression was also produced so as to predict the hydraulic conductivity at different percent bottom ash content and void ratio.	$K \left(\frac{\text{cm}}{\text{s}} \right) = \exp^{-14.2634 + 0.88735B + 13.361e}$ <p>B = percent bottom ash content e = desired void ratio</p>	For ash products. The relationship between e and hydraulic conductivity was also influenced by the amount of bottom ash incorporated in the blend. Further statistical analysis was done to verify the accuracy of the derived multiple regression equation. The equation is acceptable in predicting the vertical hydraulic conductivity given the percent bottom ash content and void ratio.
Chapuis (2004)	Chapuis (2004) suggested the modified Hazen's formula (M-H formula) by combining Hazen's original formula with the K-C formula. This is a power-law relationship that was determined by a best-fit technique	$K \left(\frac{\text{cm}}{\text{s}} \right) = 2.4622 [D_{10}^2 * e^3 / (1 + e)]^{0.7825}$ <p>D_{10} (mm) = grain size with 10% passing e = void ratio</p>	For crushed soils and rocks, the predictions of eq. were poor 0.03mm < d_{10} < 3mm 0.3 < e < 0.7
Carrier (2003)	Assuming the particle size distribution is log-linear between each pair of sieve sizes	$K \left(\frac{\text{cm}}{\text{s}} \right) = 1.99 * 10^4 (100\% / \left\{ \sum [f_i / (D_{it}^{0.404} * D_{si}^{0.595})] \right\})^2 * \left(\frac{1}{SF^2} \right) * [e^3 / (1 + e)]$ <p>f_i = fraction of soil particles between</p>	Not appropriate for clayey soils, although it will work for nonplastic silts. These conditions apply in silts, sands, and even gravelly sands. But as the pore size increases and the velocity increases, turbulent flow and the inertia term must

		<p>two sieve sizes, larger (l) and smaller (s)</p> $D_{ave\ i}$ $= \text{average particle size retained between sieves}$ $= D_{li}^{0.404} * D_{si}^{0.595}$ <p>e = void ratio</p> <p>SF = shape factor</p>	<p>be taken into account.</p> <p>But as the pore size increases and the velocity increases, turbulent flow and the inertia term must be taken into account.</p> <p>The formula is not appropriate if the particle size distribution has a long, flat tail in the fine fraction.</p> <p>The formula does not explicitly account for anisotropy.</p>
Zhang and Wang (2014)	<p>Due to the load applied to the sample, the stress field of the sample changes, which leads to the change of the pore and internal structure of the sample, so that the permeability coefficient changes. Here, the exponential function is used to represent the confining pressure and permeability coefficient. Relationship</p>	$K\left(\frac{cm}{s}\right) = k_0 e^{-\alpha \sigma_n}$ <p>k_0 = Initial permeability coefficient</p> <p>α = coefficient</p> <p>σ_n = Effective confining pressure</p>	<p>Under high confining pressure, the permeability coefficient of coarse sand and fine sand will decrease with the increase of confining pressure; the permeability coefficient of coarse sand will be within a certain confining pressure range (such as not exceeding 1 MPa in this test). After the permeability coefficient of the fine sand and the confining pressure increase, the permeability coefficient of the coarse sand gradually decreases and is smaller than the permeability coefficient of the fine sand.</p>
Wan <i>et al.</i> (2010)	<p>The relationship of the permeability coefficient K with depth can be reduced to a negative exponential model</p>	$K\left(\frac{m}{s}\right) = k_0 e^{-Az}$ <p>k_0 = Surface permeability coefficient</p> <p>A = Attenuation coefficient</p> <p>$z(m)$ = depth</p>	<p>The negative exponential model can well describe the law of aquifer permeability coefficient decay with depth, but whether it is porous or fractured medium, the model needs to be specific.</p>

<p>Fu <i>et al.</i> (2009)</p>	<p>The relationship between the pressure p and the permeability coefficient k measured by the test was imported into Origin software, and the Boltzmann model of the software was used to fit the test curve to obtain the relationship between the following pressure and permeability coefficient.</p>	$K\left(\frac{\text{cm}}{\text{s}}\right) = \frac{(A_1 - A_2)}{1 + e^{\frac{p - p_0}{\Delta x}}} + A_2$ <p>e = void ratio</p> <p>p = pressure</p> <p>A_1 = Permeability coefficient at zero load</p> <p>A_2 = Corresponding permeability coefficient for ultimate x_0</p> <p>x_0 = Corresponding pressure value when the curve has a Δx</p> <p>Δx = Slope change at each point of the relationship curve</p>	<p>Permeability coefficient is a function of the interaction between load and pore ratio, and there is no one-to-one correspondence between permeability coefficient and pore ratio. Under the action of high load pressure, although the pore ratio of soil samples with low compaction is much larger than that of soil samples with high pressure compactness, the permeability coefficient of soil samples with low pressure compactness is much smaller than that of soil samples with high pressure compactness. But for clay, the permeability coefficient of high pressure compaction is always smaller than that of low pressure compaction.</p>
<p>Ren <i>et al.</i> (2016)</p>	<p>According to the classical Kozeny-Carman approach, a new hydraulic conductivity-void ratio relationship was theoretically derived, and the Kozeny-Carman relation was proven to be a special case of the proposed equation. This equation was based on the concept of effective void ratio and the Poiseuille's law</p>	$K = C \frac{e_t^{3m+3}}{1 + e_t^{\frac{5}{3}m+1} [(1 + e_t)^{m+1} - e_t^{m+1}]^{\frac{4}{3}}}$ <p>e_t = total void ratio</p> <p>m = a positive constant for a given soil</p> $C = \frac{1}{C_F} \frac{\gamma_w}{\mu \rho_m^2} \frac{1}{S_s^2}$ <p>C_F = a dimensionless shape constant, with a value about $C_F \approx 0.2$;</p> <p>S_s (m^2/g) = the specific surface area of particles;</p> <p>γ_w = unit weight of fluid (N/m^3);</p> <p>ρ_m (kg/m^3) = particle density of soil ;</p> <p>μ ($\text{N}\cdot\text{s}/\text{m}^2$) = fluid dynamic viscosity.</p>	<p>Unlike the Kozeny-Carman equation, the proposed equation is able to predict hydraulic conductivity for fine grained soils. It presents a better capability than other models to describe measured hydraulic conductivity values of a wide range of soils, from the coarse-grained to the fine-grained.</p>

Methods based on Level of Compaction Effort

Table 3. Summary for estimating saturated hydraulic conductivity from gradation and relative compaction (RC) (Mokwa and Trimble, 2008).

General Relationship	Equation	Notes
<p>By the data evaluation and logarithmic regression, an equation based on pore parameters and the relative compaction (RC) was developed to estimate k for crushed base course materials</p>	$\ln k = \frac{1}{0.17F} \left[\frac{G_s \gamma_w}{RC \gamma_{d \max}} - 1 \right] - 10.77$ <p style="text-align: right;">cm/s</p> <p><i>F = the percent material finer than the No. 10 sieve</i></p> <p>G_s = the specific gravity</p> <p>γ_w = the unit weight of water in pcf</p> <p>$\gamma_{d \max}$ = the maximum unit weight in pcf</p> <p>RC = the relative compaction</p>	<p>Equation is useful for estimating the permeability of coarse-grained materials that are not easily measured in the laboratory and for comparing the hydraulic properties of different base course aggregates</p>

Methods based on Fines Content

Table 4. Summary for estimating saturated hydraulic conductivity from gradation ion and fines content (Bouchédid and Humphrey, 2005).

General Relationship	Equation	Notes
<p>With multivariable regression analysis, an equation was determined to estimate subbase permeability from percent fines and coefficient of uniformity. The higher the fines content and the coefficient of uniformity, the lower the measured permeability</p>	$\log(k) \left(\frac{\text{cm}}{\text{s}} \right)$ $= -2.74487 - 0.0939125F - 0.00743402C_u$ <p>F = the percent fines in percent</p> <p>C_u = coefficient of uniformity</p>	<p>The limitations on using the Equation are that aggregates should be compacted and semi-rounded particles (not angular crushed aggregates), with fines content between 3 and 14% and the coefficient of uniformity between 10 and 80</p>

Methods based on Material Type

Table 5. Summary for estimating saturated hydraulic conductivity from soil type, characterized using a soil behavior type index (Elhakim, 2016).

General Relationship	Equation	Notes
<p>(Elhakim, 2016). This equation for evaluating soil permeability based on I_c. I_c increases as the soil becomes finer. Accordingly, the soil permeability decreases as I_c increases.</p>	<p>For $1.0 < I_c \leq 3.27$, $K \left(\frac{m}{s} \right) = 10^{(0.952-3.04I_c)}$</p> <p>For $3.27 < I_c \leq 4.0$, $K \left(\frac{m}{s} \right) = 10^{(-4.52-1.37I_c)}$</p> <p>$I_c$ = The Soil Behavior Type Index</p> <p>The Soil Behavior Type Index (I_c) is determined iteratively by assuming a value of n to compute Q_{tn} that is used to calculate the corresponding I_c. Iterations are performed till the value of n reaches convergence. It has been shown that I_c increases as the soil becomes finer</p> <p>Soil permeability was estimated using the Soil Behavior Type Index (I_c) :</p> $I_c = [(3.47 - \log Q_{tn})^2 + (\log F_r + 1.22)^2]^{0.5}$ $Q_{tn} = [(q_t - \sigma_{vo}) / P_a] (P_a / \sigma'_{vo})^n$ $F_r = [f_s / (q_t - \sigma_{vo})] 100\%$ <p>q_t = CPT corrected cone resistance</p> <p>f_s = CPT sleeve friction</p> <p>σ_{vo} = in situ total vertical stress</p> <p>σ'_{vo} = in situ effective vertical stress</p> $n = 0.381(I_c) + 0.05(\sigma'_{vo} / P_a) - 0.15, \text{ where } n \leq 1.0$ <p>p_a = atmospheric pressure in same units as q_t, σ_{vo} and σ'_{vo}</p>	<p>This approach is useful in providing a detailed permeability profile with depth using CPT results. The permeability values estimated from the CPT readings are approximately half to one order of magnitude higher than the measured permeability using the falling head field test</p>

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