Mechanistic Load Restriction Decision Platform for Pavement Systems Prone to Moisture Variations

National Road Research Alliance

MnDOT Contract 1034192 Task 2: Literature Review

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CHAPTER 1: INTRODUCTION

1.1 RESEARCH PROJECT ABSTRACT AND OBJECTIVES

Excess moisture in base and subgrade soil has detrimental impacts on longevity and serviceability of pavements. Seasonal ground water level fluctuations, inundations due to storm and post-storm recess, frost penetrations and freeze-thaw effects lead to continuous moisture hysteresis and change of stress states in pavement foundation. Reliance of current empirical analysis and design procedures limit their ability to incorporate moisture-dependency and to conduct real-time and forecasted pavement capacity and load restriction analyses. This research will result in a load restriction decision platform that would assist in reliably evaluate the performance of flexible pavement systems under excessive moisture conditions. The platform would help engineers to assess pavement vulnerability to damage and, therefore, to make traffic allowance decision during and after inundation. This platform encompasses three core attributes: (1) A mechanics-based model that captures soil and base response to saturated and unsaturated soil states. It will be validated using actual field pavement tests such as MnROAD and can be further enhanced through the use of physically modelled scaled pavement sections; (2) A system-based approach to integrate impacts of various stressors (soil moisture state, vehicular loads and volume, climatic conditions etc.), current pavement conditions, subgrade properties, hydro-geology, and short-term climate forecast. Due to large number of variables and their inter-dependencies, a system dynamics modelling approach can holistically capture all significant variables and provide a user-friendly system for pavement load restriction decision making; and (3) A policy-informed decision-platform that incorporates inputs from transportation agencies and users. Such inputs will facilitate the implementation and cost-effectiveness of the proposed mechanistic approach.

1.2 SUMMARY OF RESEARCH METHODOLOGY (SCOPE)

This project will leverage systems dynamics approach to develop a mechanistic load restriction decision framework for flexible pavements. The main outcome of this project will be a toolkit for pavement engineers to make decisions regarding load restrictions due to seasonal soil moisture variations as well as during post-flooding instances. The use of system-based approach is necessary to integrate impacts of various stressors (soil moisture state, vehicular loads and volume, climatic conditions etc.), current pavement conditions, subgrade properties, hydro-geology, and short-term climate forecast. Due to very large number of variables and their inter-dependencies, a system dynamics modelling approach can holistically capture all significant variables and provide a user-friendly tool for pavement load restriction (both in current time and for future forecasting) decision making. The proposed research is divided into 10 tasks. The study starts with development of initial memo to quantify research benefits and potential implementation steps (Task 1) and literature review (Task 2), simultaneously. This will be followed with development of the system dynamics framework to mechanistically evaluate pavement load restrictions (Task 3). Thereafter, researchers will undertake Task 4 and 5 simultaneously, involving sensitivity analysis of the system dynamics model and developing a user-friendly toolkit for pavement load restriction decision process, respectively. On basis of sensitivity analysis, the system dynamics model will be refined. Using information from MnROAD (and other agency data if made available to researchers) on pavement sub-surface moisture states and pavement surface deflection (from FWD testing), researchers will calibrate and validate the tool-kit in Task 6. Task 8 will finalize the quantification of research benefits and provide guidance on implementation of the research products. Task 7 is out of state travel for researchers to present findings of this project at the annual meeting of the Transportation Research Board and Task 9 and 10 will develop and revise the final report for the study. Specific details of task activities, schedule and deliverables are described in task description section. In future, to enhance the quality and accuracy of the developed decision framework a subsequent project could include series of physically modeled and tested, scaled pavement sections.

This report serves as primary deliverable for Task-2 (literature review) of the study.

1.3 ORGANIZATION OF THE REPORT

This report is organized in seven chapters and one appendix. The subsequent six chapters provide review of the background on the individual blocks within the load restriction decision system as well as discuss pertinent literature regarding available equations and models for each of those blocks. The key blocks are determined to be:

- Effects of Excess Moisture on Pavement Performance
- Current Seasonal Load Restriction Protocols by Agencies (specifically NRRA member DOTs)
- Soil Resilient Modulus
- Soil Water Retention and Hydraulic Conductivity Models
- Water flow through Pavement Systems

Lastly, a summary is provided in chapter 7 that highlights the key findings from the literature review and briefly describes the on-going and upcoming research tasks in this study.

CHAPTER 2: EFFECTS OF EXCESS MOISTURE ON PAVEMENT PERFORMANCE

2.1 INTRODUCTION

Excess moisture in base and subgrade soils is one of the parameters that directly related to the structural capacity of pavement systems. The change in groundwater level during freeze-thaw cycles or inundations due to storm and post-storm recess and frost penetrations will cause certain amount of distress on pavement structures. In recent years, researchers showed that the subgrade materials of pavements are generally found in unsaturated condition while most of the equations used in conventional pavement design were developed based on optimum moisture content value. Also, researchers have found that the Resilient Modulus (M_R) is also highly affected by the variation of moisture content and soil suction (Yang et al. 2005, Liang et al. 2008, Cary and Zapata, 2010). These effects are important in evaluating the structural performance of pavements especially after hazardous events such as flooding. Pavements are dynamic structures and are affected by several different parameters such as climate, loading conditions, or material properties. To date, the majority of the pavement assessment models are empirical, sometimes incorporating soil index parameters or one representative moisture or suction value. Thus, a mechanistic framework that holistically incorporates all the influential factors is still needed. In the current report, various attributes that have an impact on the capacity of pavement to support vehicular traffic were explored through literature review.

2.2 MOISTURE VARIATION EFFECTS ON PAVEMENT SYSTEM RESPONSE

Vennapusa and White (2015) conducted a comprehensive post-flooding investigation of paved and unpaved roadways in Iowa; their research clearly demonstrated the need for a coupled hydro-mechanical analysis of pavement subgrade to determine the recovery of pavement to traffic bearing conditions. The FHWA Flooded Pavement Evaluation study by Sias et al. (2018) made extensive strides in development of a decision process to determine the time to opening of roadways post-flooding. A decision tree-based tool has been developed through this study (Qiao et al, 2017) that utilizes in-situ assessment procedures (such as, falling weight deflectometer) for making traffic opening decisions. The current project incorporates real-time analysis as well as future projections on the load restriction decisions along with a mechanistic analysis. These attributes were not explored in the previous flooded pavement evaluation study.

Previous researchers have looked at different parameters that influence the performance of pavement systems, and how moisture variation impacts these parameters and overall performance of the pavement system (e.g. Sultana et al. 2016). About 80% of pavement damage is reported to be directly or indirectly influenced by the presence of excess pore water pressure especially in subgrade soil (Mndawe et al. 2015) while the quality and type of base, subbase, and subgrade layers controls the overall performance of pavement structure (Santero et al. 2011, Mallick and El-Korchi 2013, Elshaer et al. 2018a).

For example, Hurricane Katrina and Rita, in 2005, resulted in extreme flooding that endangered the integrity of road pavements. Subsequently, many researchers investigated the impact of flooding on pavement deterioration (e.g. Clarke and Cosby 2007, Gaspard et al. 2007, Helali et al. 2008, Zhang et al. 2008, Vennapusa et al. 2013, Chen and Zhang 2014, Daniel et al. 2014, Khan et al. 2015, Mallick et al. 2015, Sultana et al. 2016). However, due to the lack of structural data from prior flooding, it was hard to capture the accurate degradation in pavement capacity; thus, similar systems were targeted. These researchers studied the impact of road elevations, road pavement types, and pavement thickness on the damages on roads during the first week of flooding. The results clearly indicated a loss of stiffness due to post-flooding inundation where more severe for thinner pavements (less or equal than 3 inch of asphalt layer) and pavement sections with lower stiffness (measured using Falling Weight Deflectometer) were more vulnerable to flood water damage (Helali et al. 2008, Zhang et al. 2008).

Clarke and Cosby (2007) looked at the flooded flexible pavements on State Highway 24 in McClain County, Oklahoma after the road was closed to traffic for 14 hours. They observed a 12% reduction in the Falling Weight Deflectometer (FWD) surface deflection after the road closure in comparison with the immediate post-flooding. Vennapusa et al. (2013) visited the flooded sites during Missouri River flooding in 2011, and tested the pavement shortly after water recession and again 6 to 8 months after the flooding on different types of roads at different locations. A 25-28% reduction in subgrade modulus was observed due to the flooding, 20 days after the water receded while similar numbers were reported during the 6 to 8 months post-flooding tests. Sultana et al. (2016) investigated the structural performance of pavements after January 2011 flooding in Queensland, Australia by in-situ testing within 6 weeks and 2 to 4 years post-flooding. A 25-40% reduction in FWD surface deflection and 1.5-50% reduction in Modified Structural Number (SNC) were reported while sections regained their structural strength in 4 years as a result of pavement rehabilitation procedures. Lu et al. (2017) used AASHTOWare Pavement ME to simulate extreme climatic events in Canada (including flooding). Their work demonstrated that current PavementME does not have necessary features to incorporate pavement response post-flooding as well as during events with excessive moisture contents in pavement subgrade.

In general, an increase in moisture content will result is a reduction in soil material moduli (Seed et al. 1962, Hicks and Monismith 1971, Rada and Witczak 1981, Lary and Mahoney 1984, Carmichael and Stewart 1985, Noureldin 1994, Richter 2006, Khoury and Zaman 2004, Cary and Zapata 2010). The deformation that traffic load would introduce on a pavement section is a function of soil type, porosity, of the material, and the rate of loading; thus, the deformation is at its maximum when the subgrade layer is fully saturated; i.e. complete inundation (Ovik et al. 2000). Also, the duration of inundation could result in severe loss of pavement bearing capacity, excessive permanent deformations, material degradation, and loss of bonding among different layers (Salour et al. 2015). Pavement monitoring programs such as the Long Term Pavement Performance (LTPP) that runs a Seasonal Monitoring Program (SMP) on 64 sites would be valuable tool to assess the impacts of environmental factors including temperature, moisture, and freeze-thaw cycles on pavement response (Elkins et al. 2003). Further, Amiri (2004) used a small-scale pavement section to study the "Impact of Moisture Variation of Stiffness Response of Pavements through Small Scale Models" while the moisture was controlled when the soil was compacted. Also, few researchers studied soil moisture variation effects on full scale pavement distress in HMA pavement (Saevarsdottir and Erlingsson 2013, Camacho-Garita et al. 2020).

Laval University has conducted a number of studies to evaluate the damage to flexible pavements in colder climates due to frost and excessive moisture states during spring thaw; for example Bilodeau et al. (2017) and Badiane et al. (2015). Majority of this work is conducted using a heavy vehicle simulator with a full-scale pavement test section constructed in an indoor test pit. The proposed phase-II of this research project will utilize a physical model to calibrate and refine the system dynamics-based load restriction decision process. Outcomes and data from work conducted at Laval University will be reviewed in that phase.

More recently, Elshaer (2017), as part of FHWA flooded pavement project (Sias et al. 2018) investigated the factors affecting the structural capacity of the pavement in fully saturated condition (flooded pavement), which is important when determining what factors to incorporate in the pavement model. The work also investigated different material types, thicknesses, structural numbers, and loadings. Changing moisture levels where then introduced to the pavement system through exterior environmental effects, changing subsurface water levels, and varying water table depths (Elshaer 2017, Heydinger 2003). To study the pavement response, the horizontal tensile strain at the bottom of the asphalt and the vertical compressive strain at the top of the subgrade layer were evaluated. The effects of soil suction were then considered, which lead to the incorporation of this parameter within the model developed in this research. With all of this information at hand, correlations and estimations could then be made between, the bearing capacity of the pavement in terms of short-term flooding, along with other empirical relationships that were used to estimate physical properties of the materials and stress states within the pavements, such as: resilient modulus, matric suction, poison's ratio, and structural numbers (Elshaer 2017).

Elshaer et al. (2019) used numerical modeling to study the effect of post-flooding groundwater recession on different pavement performance criteria. For example, in Figure 2-1 the effect of water table on pavement surface deflection is shown considering different pavement sections and soil types. The results emphasized the effects of water inundation on pavement structural capacity. The base course, subgrade type, and pavement structure resulted in most significant impacts on surface deflection, modified structural number, and vertical strain. However, only pavement structure had noticeable impact on fatigue performance measured by horizontal strain. Further, gradation and plasticity of unbound material played key roles in pavement structural capacity while they may behave differently in excessive water.

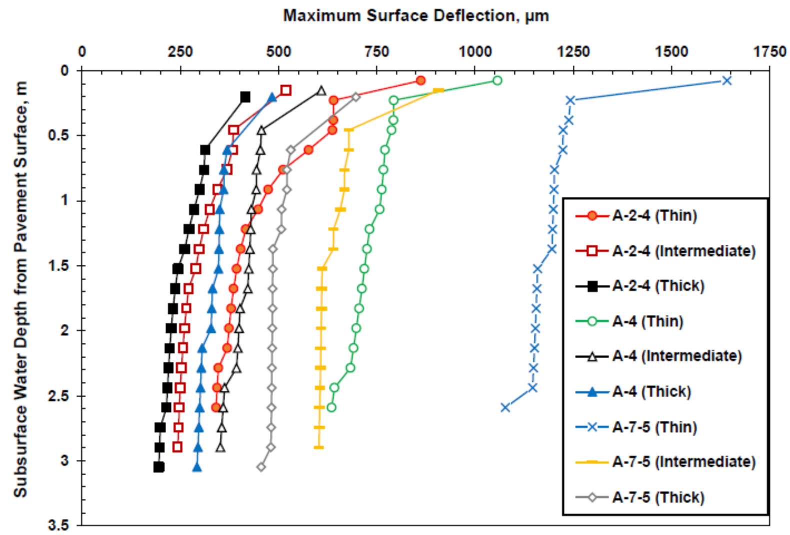

Figure 2-1 Variation of the maximum surface deflection with depth of subsurface water levels (Elshaer et al. 2019)

Further, Elshaer et al. (2017) showed how pavement bearing capacity is regained post flooding as the water recedes for different pavement section and subgrade material (Figure 2-2).

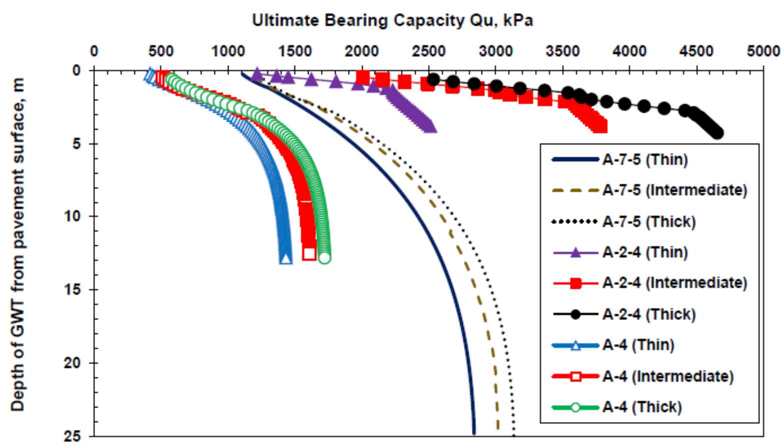

Figure 2-2 Variation of bearing capacity with groundwater table levels for the proposed soils (Elshaer et al. 2017)

The overall takeaway from these works was that the structural capacity of the pavement decreases significantly when soil is in fully saturated condition. However, the pavement may regain strength once the water dissipates and the groundwater level lowers. Another takeaway is that temperature and moisture have a significant effect on the pavement, and the influence depth for the subsurface water level is dependent on the pavement structure and material type. Finally, the material type in all layers, along with thicknesses have a significant effect on the pavements performance, and specifically the base and subgrade are the two most important factors when evaluating changes at the bottom of the asphalt layer, which is a major reference location for determining distresses within the pavement system (Thom & Brown, 1987).

2.3 PERTINENT INPUTS AND MODELS TO ASSESS PAVEMENT CAPACITY IN CONTEXT OF EXCESS MOISTURE STATES

2.3.1 Pavement Structure and Condition

Pavements are multi-layered structures. Transfer of heavy tire loads to subgrade in flexible pavements rely on concept of load distribution. Whereby the stress levels from top of pavement structure continually decreases until it reaches levels safe for subgrade to carry. Most common configuration of flexible pavement includes asphalt layers, base layer, subbase layer, prepared subgrade and natural subgrade. The role of each of these layers is different in the pavement structure and their sensitivities to changes on moisture level also vary.

Asphalt layers near the top of pavement structure often comprise of multiple lifts. The thicknesses of these layers depend on the anticipated traffic levels. Usually the wear course or top-most lift is constructed with more angular aggregates due to very high tire pressures. Wear course is also usually specified with a smaller maximum aggregate size to ensure smoother pavement surface and to increase durability. The non-wear courses experience lower compressive stresses, but often undergo greater tire induced tensile stresses and strain due to flexure of the pavement system under traffic loading. Asphalt layers are sensitive to moisture and often during the design of these materials, testing is conducted to determine moisture damage susceptibility of asphalt mixtures. Typically tests such as modified Lottman test (AASHTO T-283) and Hamburg Wheel Tracking test (AASHTO T 324) are used. In the present research the moisture damage susceptibility of asphalt mixtures is not planned to be considered within the load restriction evaluation. The current asphalt mixture specifications used by NRRA partner agencies all require testing for moisture susceptibility as part of the mix design process. Hence, the asphalt mixtures from partner agencies are expected to have minimal moisture induced damage potential during the periods of excessive pavement moisture state.

The base and subbase courses in flexible pavements provide economical layers that not only contribute in stress distribution but also provide lateral drainage to the structure. These layers help in movement of water away from the roadway foundation into drainage ditches. In colder climate regions, these layers also help lower the extent of frost penetration into soil subgrade. Since majority of base and subbase courses are constructed with natural and processed aggregates, their mechanical properties vary significantly with moisture content. The current MnPAVE system provides a good reference for seasonal adjustment to base and subbase layer resilient modulus. The AASHTOWare Pavement ME also incorporates the moisture content of base and subbase layers in determining the modulus of these layers which are then used in the pavement response analysis. Previous research by Elshaer et al. (2019) and Knott (2019) have successfully incorporated impacts of moisture level variations in base and subbase layers into adjustments to these layers' resilient moduli. Similar approaches are planned to be adopted in the proposed research. Hydraulic conductivities of these layers will also be incorporated into the proposed system dynamics framework, since the framework is anticipated to include hydraulic analysis for continuous prediction of moisture movement through the pavement structure.

The prepared and natural subgrade properties (stiffness, soil moisture retention as well as hydraulic conductivity) will be critical in the proposed analysis. Later chapters provide substantial insight into the existing literature and relationships that have been developed.

The knowledge of the pavement cross-section (number of layers, their thicknesses and materials types) will be important input to the load restriction decision system. In absence of known cross-section, several typical sections will also be included in the decision toolkit. Use of typical cross-sections will lower reliability of predictions, users will be made aware of this aspect.

The structural condition of the roadway also plays an important role in terms of its load bearing capacity. Due to structural distresses from traffic loads and climatic stressors, the load bearing capacity of roadway often reduces with increasing time (and traffic). Typical pavement design and analysis models (such as, MnPAVE or Pavement ME) adopt miner's hypothesis, whereby the damage functions expressing structural distresses are assumed to be cumulative in nature. Since the proposed research is not focused on life-time simulation of pavement capacity and will be focused only on durations when there is excess moisture within pavement, the pavement condition will be incorporated in the determination of the pavement structural response (such as surface deflection). The pavement condition is anticipated to be estimated from the remaining service life provided by pavement management systems.

2.3.2 Moisture-Dependent Soil Properties

As discussed in Section 2.2, excessive moisture in pavement systems especially in subgrade soils will reduce the pavement foundation capacity and result in surface deflection and cracking. This has been shown through numerical modeling (e.g. Elshaer 2017, Haider and Masud 2018), physical small scale and full scale modeling (e.g. Amiri 2004, Saevarsdottir and Erlingsson 2013, Camacho-Garita et al. 2020), and field performance assessment (e.g. Clarke and Cosby 2007, Helali et al. 2008, Zhang et al. 2008, Sultana et al. 2016). Soil properties play a key role in pavement response; thus, accurate assessment of these properties under various degree of water saturation is crucial.

Subgrade soil resilient modulus is probably the most influential factor that controls the overall stiffness of the pavement systems. Developing moisture-dependent resilient modulus has been in the forefront of transportation geotechnics research. Especially, with the advance of unsaturated soil mechanics, significant efforts have been made to correlate soil suction and state of stress to resilient modulus in a more mechanistic setting. This is important as vehicular traffic imposes changes in pore pressure or suction in soils. This emphasizes the need for such suction-dependent modulus models that can capture the transient pavement response. CHAPTER 4 is devoted to the review of the resilient modulus models and equations. The goal is to develop a set of relatively well-characterized formulations for different soils and applications that can be used in the proposed system dynamics framework and eventually within the load restriction toolkit.

Suction is proven to be the factor that changes the stress state and impact the soil behavior. However, soil moisture in either gravimetric or volumetric forms are often being measured in the field. Therefore, it is important that these two soil properties be accurately correlated, so they can be interchangeably used in models. Three major soil water retention models are introduced in CHAPTER 5, where their strengths and weaknesses are discussed. In addition, commonly applied unsaturated hydraulic conductivity models are presented and discussed. It is expected that one or two of the models may be implemented within the system dynamics framework for both estimating soil properties and hydraulic analysis.

2.3.3 Climatic Factors

It is well-known in pavement engineering that the performance and lifespan of pavements are impacted severely by climatic factors. Specifically, temperature and moisture are major stressors for pavements (Huang 2012; Mallick and El-Korchi 2013). Recent researches have focused extensively on adapting roadway networks for greater resiliency against changing climatic conditions (Knott, 2019; Pregnolato, Ford, Wilkinson, and Dawson, 2017; EPA, 2017). The link to how this changing climate is affecting the groundwater variation, and therefore affecting the pavements performance, will be an important factor to consider when developing the system dynamics model.

Development of climate projections models is not within scope of this study. Researchers will adopt use of existing climatic forecast data within the system dynamics framework. At this point in time, the short-term meteorological forecast (7 and 14 day) from National Oceanic and Atmospheric Administration (NOAA) will be utilized. The precipitation forecast from this source will be used as user input to the pavement hydraulic analysis to obtain the saturation profiles within pavement structure.

2.3.4 Ground Water Flow Models

Seasonal fluctuation of ground water level or water movement through the unsaturated soils during flooding both would impact the soil moisture/suction profile in depth, which in turn, impact the soil properties and overall pavement response. Thus, a well-designed, mechanistic pavement response assessment protocol requires a robust hydraulic analysis of water flow through pavement layers. A review of available approaches and past research is presented in CHAPTER 6. These methods range from complex analytical solutions to simplified approaches and from numerical models to more empirical formulations. The target problem in this research is to track the groundwater level and, as a result, the moisture profile in depth during and after flooding when the water level recedes. In addition, it is planned to incorporate the climatic inputs into these models. Further, the goal is to have a model that is reasonably accurate, yet simple for practical applications.

2.3.5 Pavement Structural Response Model and Capacity Indicator(s)

Historically, different methods have been proposed to analyze the structural performance of pavement systems. The early models made simple assumptions about the loading and the layered system, while more recent models incorporate more complex soil response and can simulate multi-layer systems. Some of the common approaches are single-layer elastic theory, multi-layer elastic theory, finite element methods, viscoelastic theory, dynamic analysis, thermal models, and nonlinear plastic behavior models.

Single-layer and multi-layer elastic theory are based on fundamental formulations of mechanics of materials, such as Hooke's theory of elasticity. Over time, different assumptions and changes, for either loading or the layered system, were made in these models which made them more accurate. The evolution of these models follows this timeline: half-space space under a point load (Boussinesq, 1885), semi-infinite space due to a circular load (Newark, 1942; Foster & Ahlvin, 1954; Ahlvin & Ulery, 1962), two layers due to a circular load (Burmister, 1943), three-layer systems (Jones, 1962), and finally multilayer systems and finite element models.

The use of multilayer analysis, specifically layered elastic analysis, is current state of practice in the majority of flexible pavement analysis and design systems (such as, MnPAVE, Pavement ME, CalME etc.). In the proposed framework, the pavement surface deflection will be considered as an indicator of the pavement capacity. Furthermore, use of system dynamics for sensitivity analysis and real-time evaluation requires usage of a closed-form solution. Another challenge in evaluating a multi-layer system is the incorporation of moisture variation in depth. Elshaer et al. (2018 a,b) discussed the impact of the modulus equation option and also the selected approach to incorporate variable moisture on pavement response. This included the choice of suction- versus degree of saturation-dependent equation, inclusion of multi-layer subgrade with variable moisture versus incorporating a representative effective moisture content value, and the choice of empirical versus more mechanistic resilient modulus functions. These effects were tested for different soil types and pavement structure. The results indicated the simple models especially for non-plastic soils might be sufficient as long as the depth of stress influence and the effective moisture content is considered. The results were verified against FWD data recorded from the Long-Term Pavement Performance (LTPP) records.

For the load restriction decision system, use of two and three-layer solutions will be adopted in the system dynamics framework. Comparative evaluations will be undertaken to ensure that the results of these solutions are in agreement with multilayer analysis program (such as, WESLEA or JULEA).

2.3.6 Traffic Loads

When determining what traffic load to use as an input parameter in the load restriction decision system, a few different parameters should be considered. The key parameters in all load cases are the vehicle types (tire and axle configurations, pressure etc.), loading repetitions for each type, and future projections of both loading and repetitions. This is done by observation and quantification of the traffic traveling over a given roadway. Then, past data can be looked at, and with other future estimates, the future traffic on the roadway can be estimated.

There are two approaches to determine the expected loads on the given pavement over its entire design life. One approach is to convert all magnitudes of loading and repetitions of loading to an equivalent unit using approaches such as equivalent damage, a commonly used example for this is equivalent single axial load (ESAL). The other approach is to use a load spectrum, which characterizes loads directly by number of axles, configuration, and weight. This method is typically more complex since the structural analysis requires use of each vehicular combination to be evaluated to obtain relevant responses. Both methods follow typical equations and/or procedures that have been well laid out (such as, AASHTO, 1993 and FHWA, 2019).

For the proposed load restriction decision framework, the use of ESAL approach is not appropriate, since the damage potential from each vehicle type needs to be evaluated. Furthermore, a unique feature of this research is to provide users with a vehicle-class specific load restriction decision. Thus, in this study vehicle class-based traffic inputs will be utilized. At present, the 13-category FHWA vehicle classification will be adopted (FHWA, 2014). The pavement response analysis will be conducted at median, 75th percentile and 90th percentile load levels for each vehicle class. The nationally applicable load level distributions for each vehicle class are provided in the AASHTOWare Pavement ME system. The higher percentile loads will be adopted on the basis of the criticality of the roadway in question.

2.3.7 Other Variables

In addition to the parameters that were previously mentioned, there are a few other parameters that can potentially impact the performance of the pavement system. Physical properties of the materials used within the pavement system can affect the overall performance of the system, which was previously mentioned (Huang, 2012). However, some additional soil parameters that have the potential to affect the overall performance of the pavement system could be friction angle and cohesion values of certain soils. Also, other parameters of cohesive soil could have an effect on the performance of the pavement system, such as both liquid limit and over-consolidation ratio. Compaction and consolidation characteristics themselves should be considered for all materials and soils used within the pavement system, since they both have a direct relationship in terms of both strength and drainage within the pavement system (Holtz, 2011).

Another consideration that should be made that would have an effect on the pavement performance is the surrounding environment. For example, if the pavement system is located next to an ocean, a changing climate or flooding events may lead to the pavement system being exposed to more moisture than other systems (FHWA2019). If the surrounding terrain tends to drain additional water into the pavement, this would also expose this pavement to more moisture than other systems. In conjugation with this consideration, the direction of the flow of water should also be considered as an important factor that may affect the overall performance of the pavement system.

Construction considerations should be made as an important factor that may affect the overall performance of the pavement system (Huang, 2012). For example, if the compaction requirements that were called for and accounted for in the model, were not what was implemented in the actual pavement system, then this would affect the overall performance of the pavement in a negative way.

2.4 SUMMARY

A well-designed and user-friendly load restriction decision platform would heavily rely on 1) an accurate assessment of pavement response under excessive and fluctuating water in base, subbase, and subgrade layers; 2) a mechanistic evaluation of pavement performance that can holistically incorporated several key influential factors; 3) the capacity of the platform to prioritize the impactful factors in the response analysis and rank the load restriction recommendation; 4) the ease of access by users with different expertise and input data.

The project will leverage system dynamics sensitivity analysis and statistical approach to develop a load restriction decision protocol that can meet the above qualifications. The analysis will be based on the list of expected key players in pavement response analysis during and after inundation. The following chapters discuss the history and state-of-the-art in some of these influential factors that are considerably sensitive to amount of moisture.

CHAPTER 3: LOAD RESTRICTION PROTOCOLS

3.1 WHY ARE LOAD RESTRICTIONS NEEDED?

Seasonal road restrictions are weight limits that are enforced by various state's Department of Transportation (DOT) as well as local highway agencies. These limitations are put in place in order to reduce the amount of damage that a certain roadway will experience. These restrictions are put in place when the pavement system is most vulnerable to experience damage; this usually occurs when the frost from the winter season thaws into the spring season or after inundation due to flooding. This results in excessive water within the pavement system itself, causing a weaker system. The load limitations are then removed when the roadway is able to carry legal traffic weight without accelerated damage to the structure. Each road load limitation varies from state to state and depends on a number of various parameters such as loading scenarios, local temperature, and roadway types and conditions.

Eight National Road Research Alliance (NRRA) states DOTs (California, Illinois, Iowa, Michigan, Minnesota, Missouri, North Dakota, and Wisconsin) were investigated to understand how agencies are already evaluating load restrictions on given roadways and how road closures and opening decisions are made. Some of these parameters include, but not limited to frost depth, temperature forecasts, and pavement strength. Looking at current polices of different agencies allows to establish a baseline to determine what important information are needed when setting a load restriction. If a certain factor, such as temperature, was repeated throughout multiple NRRA states, it was then noted that the factor should be used when investigating whether a road should have a load restriction. Creating this baseline for important factors in setting a load restriction will be useful when incorporating different factors into the proposed model. Load restriction state specific data was successfully found and collected for the following NRRA states: Minnesota, North Dakota, and Wisconsin. The following states did not have information publicly available: California, Iowa, Missouri, Michigan, and Illinois. Although the specific limits and restrictions were not listed for these states, similar factors were discussed when determining load restrictions for roadways.

3.2 MINNESOTA DOT LOAD RESTRICTIONS

Minnesota is also a NRRA state that uses four different factors to determine whether a roadway should have a load restriction. These four factors include: daily temperature forecasts, future temperature forecasts, a parameter called the cumulative thawing index, and the depth at which the frost is located below the ground. The table below (Table 3-1) shows each factor in the guideline along with some specific notes and limitations used by the Minnesota Department of Transportation (MnDOT) when setting or lifting a load restriction. (Minnesota Department of Transportation Engineering Services Division, 2014).

Table 3-1: MnDOT Load Restriction Factors

MnDOT	
Factor in Posting of Load Restriction	**Notes**
Temperature	Load restrictions will be scheduled when the 3-Day weather forecast indicates (CTI) will exceed 25F degree-days and longer-range forecasts predict continued warmth
Cumulative Thawing Index (CTI)	Used in conjunction with temperature forecasts to set and lift load restrictions within each different frost zone
Frost Depths	With other key parameters located at each frost zone, this help place an end date to the load restriction
Forecast Daily Air Temperature	

A major parameter that MnDOT uses is called the cumulative thawing index (CTI). This index represents a running total of each day's thawing index that starts from a value of zero degrees Fahrenheit during the winter freeze. The daily thawing index is the amount the daily average temperature is above the reference temperature for that day, and the reference temperature is based on the monthly average temperature. The department has a specific set of rules, limitations, examples, and equations when dealing with CTI (Chiglo 2014).

As (Table 3-1) describes MnDOT uses both daily and future forecasted temperatures to help determine if a road requires a load restriction enforced on it. This observed temperature forecast is then used along with the CTI to determine load restrictions. The department specifically looks for the time when the three-day weather forecast indicates that the CTI for a specific frost zone will exceed 25 degree-days and also that no long-range temperature forecast predicts for warm days to come. If this condition holds true, then restrictions will be scheduled and the advance notice for the public will be released.

The next factor used in determining load restrictions is the depth of the frost table under the ground surface. This is important because it indicates at what level the frozen water table is located near the ground surface, and if too close to the surface that would be a cause for concern. The speed at which this frost table lowers, and thaws depends on several factors such as; depth, soil moisture content, and spring weather patterns (Guthrie et al., 2016). All these parameters can vary from year to year, therefore the load restrictions in each year will also vary and will depend heavily on the past experience of the department

3.3 NORTH DAKOTA DOT LOAD RESTRICTIONS

North Dakota's DOT (NDDOT) uses four different factors to determine whether or not a certain road should have a load restriction posted or not. These factors include; current temperature in the roadway, temperature forecasts, current strength of the roadway, and previous experience. The following table (Table 3-2) describes each factor in more detail along with any notes and or limitations that apply to each (North Dakota Department of Transportation, 2019).

Table 3-2: NDDOT Load Restriction Factors

NDDOT	
Factor in Posting of Load Restriction	**Notes**
Temperature in Base Layer	Probes put into base layer of pavement section. When approach 32F, planning of posting begins
Long Range Temperature Forecast	When indicate low temp. approaches freezing point and the daily highs are in the upper 30's or 40's, restrictions are planned
Falling Weight Deflectometer (FWD)	Measures strength of roadway bases. Used for both initiating and lifting load restrictions in combination with long range forecasts and area wide moisture conditions
Past Experience	Most significant damage occurs during first 4 weeks after spring thaw. Lead to close monitoring of weather forecasts and sub-base temps.

The first factor used by NDDOT when determining if a certain roadway needs a load restriction is the temperature within the base layer. Temperature probes are placed within the base of different roadway systems throughout the state. The temperature is then observed and recorded for each section of roadway. NDDOT begins planning the posting of load restrictions when the temperature in the base layer begins to approach 32 degrees Fahrenheit. The next factor used by NDDOT is the long-range temperature forecast within each region. When the department observes consistent forecasted daily temperatures that have daily highs in the range of upper 30's or 40's (degrees Fahrenheit), load restrictions are then planned. The next factor that the NDDOT uses when determining whether a roadway needs a load restriction is the actual strength of the particular roadway. The way this strength is measured is by using Falling Weight Deflectometer (FWD). This allows for measuring both the strength of the roadway base along with the strength of the asphalt surface. The data collected from FWD along with long range weather forecasts and moisture conditions over the whole area, provide the basic information needed for NDDOT to both initiate and lift a load restriction on a given roadway.

The last factor, and most significant factor used by NDDOT when determining load restrictions is past data and experience (North Dakota Department of Transportation, 2019). Using past information, the department has been able to limit the time frame to when the most significant damage is seen on a given roadway. NDDOT recognized that the most significant pavement damage occurs during the first four weeks after the onset of spring thaw. This allows for closer monitoring of weather forecasts and sub-base temperatures during this time to either enforce load restrictions or lift them in a shorter time frame.

3.4 WISCONSIN DOT LOAD RESTRICTIONS

Wisconsin's department of transportation (WisDOT) is the third NRRA state that describes how load restrictions are set for specific roadways within the state. WisDOT uses five different parameters when determining if a road should have a load restriction enforced or if a restriction should be lifted, which can be seen below in (Table 3-3). The five factors are the following: temperature forecasts, the depth at which the frost level is below the ground, visual inspection, axle configuration along with vehicle weight, and the trip type that the vehicle driving over the specific roadway is taking (Wisconsin Department of Transportation, 2018).

Table 3-3: WisDOT Load Restriction Factors

WisDOT	
Factor in Posting of Load Restriction	**Notes**
Rising Temperatures	Use weather forecasts and Cornell Pavement Frost Model (CPFM) (Miller et al. 2015) to estimate when restrictions are needed
Frost Tube Readings	Frost tube should be checked on Mondays and Thursday and reported to BHM until seasonal posted roads restrictions are declared in each zone
Frost Depth	Reaches 6 inches below pavement surface
Road Level	Seasonal Posted Roads cannot be declared until Class II road restrictions are declared, and Seasonal Posted Roads must end before Class II road restrictions are ended
Maintenance	Weekly monitoring for weeping and pumping, advising Bureau of Highway Maintenance Freight Engineer when Seasonal Postings shall end for each zone
Axle Configuration and Vehicle Weight	Posted limits are normally 6 tons per single-axle and 10 tons for any 2 axles less than 8’ apart Gross vehicle weight or combo of group axles is 24 tons
Trip Type	All single trip and most annual overweight travel is not permitted during this time on seasonal posted road sections

WisDOT uses future and daily weather forecasts in conjunction with MnDOT’s frost model in order to set and lift certain roadway load restrictions. The next parameter that the department considers when enforcing load restrictions is the depth at which the frost table is located beneath the ground surface. This is done by using frost tubes that are placed into a hole with undisturbed and uncompacted soil. The readings are reported two times a week until the frost level is no longer six inches from the pavement surface, which is the limiting distance for setting the load restriction (Wisconsin Department of Transportation, 2018). Another factor used by WisDOT is visual inspection during maintenance of a specific roadway. Weekly monitoring is mandatory with the intention of looking for signs of weeping and pumping within the roadway. If signs of these conditions are shown, it must be brought to the attention of the Bureau of Highway Maintenance (Bureau of Highway Maintenance WisDOT, 2019) where then a restriction will be enforced, or if these conditions are no longer occurring for a steady amount of time then a restriction can be lifted.

The department also limits the axle configuration and vehicle type when enforcing a load restriction. This is to ensure that the ultimate strength of the roadway will not be impaired if overloaded during its weaker timeframe. WisDOT has certain expectations and procedures for this limitation that are outlined on the DOT’s website (Wisconsin Department of Transportation, 2018).

3.5 IMPORTANT FACTORS CONSIDERED AND TRENDS FOUND

Looking at the three NRRA states that had available information for how load restrictions are set and lifted, certain factors and parameters showed up multiple times across the three states. The major factor used in all three states was looking at temperature forecasts for both the future along with current conditions. The next factor that is being consistently used is the depth of the frost layer. Along with this factor, there was a stress to focus on how the water drained with the current soil conditions once the frost layer thawed. The last factor that constantly was used in all three NRRA states was setting a certain axial configuration, trip type, and vehicle weight which is the basis for the load restriction. Further, past experience is also commonly considered in load restriction decisions.

In this research, these factors will be implemented in a decision model, where the model will encompass the following breakthroughs:

- A mechanistic load restriction protocol will be developed that include an analytical or empirical hydro-mechanical analysis.
- The effect of moisture variability is investigated through a holistic sensitivity analysis.
- The load restriction protocols will be extended to flooded zones where water in inundated zones recedes and the load restriction can be lifted.

CHAPTER 4: RESILIENT MODULUS

4.1 INTRODUCTION

The effects of moisture on soil's resilient modulus have been investigated by many researchers in the past (e.g. Sauer and Monismith 1968, Edris and Lytton 1976, Fredlund and Morgenstern 1977, Noureldin 1994, Drumm et al. 1997, Ceratti et al. 2004, Yang et al. 2008, Khoury and Khoury 2009, Sawangsuriya et al. 2009, Khoury et al. 2010, Cary and Zapata 2010, Han and Vanapalli 2015). As a result of these investigations several analytical and empirical models have been proposed to estimate the resilient modulus of subgrade soil under various moisture and stress states; some being simple and applied and some being complex and mechanistic (e.g. Seed et al. 1967, Moossazadeh and Witczak 1981, Witczak and Uzan 1988, Witczak et al. 2000, Khoury and Zaman 2004, Yang et al. 2005, Liang et al. 2008, Cary and Zapata 2010 and 2011, Sivakumar et al. 2013, Khosravifar et al. 2015). A master list of several resilient modulus prediction equations has been developed and a subset of this list is provided in the APPENDIX.

To date, the most commonly used equation is the extended version of MEPDG equation for resilient modulus at optimum water content from the results of extensive experimental material evaluation (Zapata et al. 2007). In this method, Equation 4-1, is used to adjust the estimated resilient modulus at optimum water content using Equation 4-2, based on the degree of water saturation.

$$\boldsymbol{log\left(\frac{M_R}{M_{R-OPT}}\right) = a + \frac{b-a}{1+exp\left[ln\left(-\frac{b}{a}\right)+k_m(S-S_{OPT})\right]}}$$

Equation 4-1

$$\boldsymbol{M_{R-OPT} = k_1 p_a \left(\frac{\theta_b}{p_a}\right)^{k_2}\left(\frac{\tau_{oct}}{p_a}+1\right)^{k_3}}$$

Equation 4-2

where S = degree of saturation (in decimals); S_{OPT} = degree of saturation at optimal water content (in decimals); a = minimum of log (M_R/$M_{R\text{-}OPT}$); b = maximum of log log (MR/MR-OPT); and km = regression parameter. Parameter values a = −0.5934, b = 0.4, and km = 6.1324 are suggested for fine-grained soils, and parameter values a = −0.3123, b = 0.3, and km = 6.8157 are suggested for coarse-grained soils. Also, where p_a = atmosphere pressure (i.e., 101.3 kPa), θ_b = bulk stress, τ_{oct} = octahedral shear stress; and k1, k2, and k3 = model parameters.

Due to the advance of the mechanics of unsaturated soils, more mechanistic equations were proposed incorporating the state of stress and soil suction. Han and Vanapalli (2016) reported a summary of a suite of these equations to estimate or predict suction or moisture-dependent resilient modulus for pavement base-course and subgrade soils. These equations were broken up into three categories: empirical relationships, constitutive models incorporating the soil suction into applied shearing or confining stresses, and constitutive models extending the independent stress state variable approach.

After reviewing these predictive equations, representative formulas are selected based on the target soil type recommended and the model's performance in predicting the resilient modulus values. Soil types were broken into three different categories according to AASHTO classification: A-1 soils, A-2 and A-3 soils, and A-4 through A-7 soils (AASHTO, 1993). The corresponding soil types by the Unified Soil Classification (UCS) system definitions are; (GP, GW, SP-SM, SM, and SP) for A-1 soils, (GM or GC, SM, SC, and SM-SC) for A-2 and A-3 soils, and (CH, CL, CL-ML, MH, ML, SC, SM, SM-SC, A-6) for A-4 through A-7 soils (Natural Resources Conservation Service, 2019).

4.2 RESILIENT MODULUS FOR A-1 SOIL TYPES

The research that was performed on the Canadian Long-Term Performance Project (C-LTPP) resulted in a generalized model that quantifies the modulus-water sensitivity of typical base material (Doucet & Dore' 2004). The proposed model is an empirical relationship developed through resilient modulus tests on several partially crushed and crushed granular materials, seen below in Equation 4-3 and Equation 4-4.

$$\Delta \boldsymbol{M_R} = \boldsymbol{-8700(u_a - u_w) - 17,000}$$

Equation 4-3

$$\boldsymbol{M_R = 1060\theta_b - 8700\psi + 57000}$$

Equation 4-4

The model uses the matric suction (ψ) (kPa) to describe the modulus water sensitivity, and the balance between air pressure (u_a) and pore water pressure (u_w). Bulk stress is defined as (θ_b), (ΔM_R) is the variation in resilient modulus in kPa, and (M_R) is the resilient modulus also expressed in kPa.

Further, research was conducted to develop correction factors (CF) under the National Cooperative Highway Research Program (NCHRP). M-EPDG testing methods were performed in order to obtain an in-situ M_R for underlying materials (MEPDG, 2004). This value was then compared to a laboratory M_R value, and a correction factor was then developed. Other researchers also participated in the effort to establish the CF of base material, while taking into account stress state and in-situ moisture content for Florida granular materials (Oh et al. 2012). Laboratory test data on resilient modulus and soil suction, the relationship between resilient modulus, stress state, and moisture content was investigated. After reviewing the resilient modulus from the M-EPDG (MEPDG, 2004) the following equation was developed, presented in Equation 4-5; where bulk stress is modeled by θ, the octahedral shear stress is τ_{oct}, ψ is suction, and P_a is atmospheric pressure (100kPa).

$$\boldsymbol{M_R = k_1 P_a \left(\frac{\theta_b + 3k_4\psi\theta}{P_a}\right)^{k_2} \left(\frac{\tau_{oct}}{P_a} + 1\right)^{k_3}}$$

Equation 4-5

The equation also includes four regression constants, k_1 through k_4. One constant specifically, k_4, accounts for the effect of moisture content variation on the bulk stress term.

Through a micromechanical approach to model partially saturated soils, a relationship between the mean principal stress acting on the system and the Helmholtz free energy per unit initial volume was developed (Lamborn 1986). From this equation, a relationship between the mean principal stress and the change in soil suction was developed (Chandra et al. 1989). Based on this relationship, Equation 4-6 was proposed, which is similar to Equation 4-5; in which S is suction and V_w is the volumetric warer content.

$$\boldsymbol{M_R = k_1 P_a (\frac{\theta_b + 3k_4 S V_w}{P_a})^{k_2} (\frac{\tau_{oct}}{P_a} + 1)^{k_3}}$$

Equation 4-6

The mean principal stress is known to be one third of the bulk stress; this means the change in bulk stress due to soils suction can then be calculated. The change in soil suction equates to an additional confinement being imparted, which is then added to the bulk stress associated with surface loads and gravimetric stresses.

4.3 RESILIENT MODULUS FOR A-2 AND A-3 SOIL TYPES

Cary and Zapata (2011) studied the effect of moisture variation on the resilient modulus of unbound materials in a pavement structure. Earlier, there were models developed that attempted to capture how moisture variation affected a pavement structure. However, these models were based on a total stress analysis and were mostly empirical. The goal in Cary and Zapata's research was to understand the relationship between pore water pressure and the resilient modulus response. They introduced matric suction, which is a fundamental stress variable, as a predictive variable in the Universal Model adopted by the M-EPDG (MEPDG, 2004).

Testing was done on a Triaxial system that allowed full control and measurement of pore water and pore air pressures. The system was also capable of simulating both drained and undrained conditions, in this study both conditions were tested, and resilient modulus was measured. The materials tested in this study were a typical granular base material found in Arizona and a subgrade material that was commonly found in the Phoenix Valley and was classified to be a clayey sand.

After testing, successful modeling of the effects of suction on modulus resulted in a smooth transition from unsaturated soil conditions to saturated conditions. This led to modifications being made to the Universal Model, and gave the proposed equation shown below; i.e. Equation 4-7.

$$\boldsymbol{M_R = k'_1 * P_a * \left(\frac{\theta_{net} - 3 * \Delta u_{w-sat}}{P_a}\right)^{k'_2} * \left(\frac{\tau_{oct}}{P_a} + 1\right)^{k'_3} * (\frac{(\psi_{m_0} - \psi_m)}{P_a} + 1)^{k'_4}}$$

Equation 4-7

Where, P_a = atmospheric pressure, k'_1 through k'_4 are regression constants that depend on material type, $\theta_{net} = \theta - 3u_a$, net bulk stress and u_a is pore air pressure, Δu_{w-sat} = build-up of pore water pressure under saturated conditions, in such cases $\Delta\psi_m = 0$, τ_{oct} = octahedral shear stress, $\psi_{m_0} =$

initial matrix soil suction, and $\Delta\psi_m$ = relative change of matric soil suction with respect to ψ_{m_0} due to build-up of pore water pressure under saturated conditions, in this case $\Delta u_{w-sat} = 0$.

Net bulk stress is used over bulk stress to accommodate for modeling the transition between unsaturated and saturated soil states. This is because as the condition happens, pore air pressure will approach zero, and the net bulk stress will approach the bulk stress again. The third factor is the new term that makes this model different from the Universal Model. This is the term that attempts to capture the contribution of matric suction in the overall resilient response of the material under saturated, undrained conditions. These conditions are in effect when the relative change of matric suction approaches zero. The actual matric suction at the time of the resilient modulus measurement can be obtained by subtracting the relative change in matric suction from the initial matric suction.

The plus one term after the octahedral shear stress is to avoid the same problem of constants approaching zero. This term is also normalized by the atmospheric pressure, and therefore keeps the regression constants non-dimensional. Once this term approaches one, the model allows for saturated conditions, and therefore matrix suction doesn't contribute to resilient modulus anymore. When the excess pore water pressure becomes equal to the external applied loads, the effective stress in the material approaches zero. The regression constants were obtained for each of the base and subgrade material by plotting the predicted resilient modulus against the measured resilient modulus.

In another study by Sahin et al. (2013) Equation 4-8 was developed for granular bases. This model is based on micromechanics theory and thermodynamics laws.

$$\boldsymbol{M_R = k_1 P_a \left[\frac{\theta_b - 3f\theta\left(\psi_0 + \beta\frac{\theta_b}{3} + \alpha\tau_{oct}\right)}{P_a}\right]^{k_2} \left(\frac{\tau_{oct}}{P_a} + 1\right)^{k_3}}$$

Equation 4-8

Where; k_1 through k_4 are model parameters, τ_{oct} = octahedral shear stress, ψ_0 = initial matrix soil suction, P_a is atmospheric pressure, θ_b is bulk stress, θ = volumetric water content, f = saturation factor ($1 < f < \frac{1}{\theta}$), and α and β are Henkel pore-water pressure parameters.

4.4 RESILIENT MODULUS FOR A-4 THROUGH A-7 SOIL TYPES

Yang et al. (2005) and Ng et al. (2013) proposed equations that deal with A-4 through A-7 soil types. For example, when dealing with A-4 through A-7 soil types, Equation 4-9 was developed by Yang et al. (2005). The soils examined in this research project consisted of two fine-grained subgrade soils (one A-7-5 soil and one A-7-6 soil) from Taiwan, China, over the soil suction range of 0–10,000 kPa.

$$\boldsymbol{M_R = k_1(\sigma_d + X^{\psi})^{k_2}}$$

Equation 4-9

where, σ_d = deviator stress, X = Bishop's effective stress parameter, ψ = soil suction, and k_1 and k_2 are model regression parameters. The regression parameter values depend on the material being used. For the A-7-5 soil type $k_1 = 274.2$ and $k_2 = 1.24$, and for the A-7-6 soil type $k_1 = 111.5$ and $k_2 = 1.27$.

This equation uses a single model parameter (k_2), derived from regression studies, to predict the behavior of the resilient modulus with respect to both matric suction and deviator stress. Positive k_2 values indicate that resilient modulus increases with both matric suction and deviator stress, while negative values do the opposite and show a negative trend in resilient modulus. It is documented through this study that this model reasonably captures the behavior in resilient modulus with respect to matric suction.

The effect of moisture on resilient modulus of a material, through a suction-controlled triaxial apparatus was studied by Ng et al. 2013. The material tested was a decomposed tuff material that was collected in Hong Kong. It can be classified as a silt (ML) by USCS or as A-7-6 soil type by AASHTO (AASHTO, 1993). In this research, the effect of two stress-state variables (matric suction and net stress) along with the wetting and drying history and how it affected the resilient modulus of the material were investigated. Also, they studied the effect of load repetitions on the material. The results yielded Equation 4-10 for the resilient modulus of a material under both saturated and unsaturated conditions.

$$\boldsymbol{M_R = M_0\left(\frac{\theta_{net}}{\theta_{ref}}\right)^{k_1}\left(\frac{q_{cyc}}{\theta_{ref}}\right)^{k_2}(1+\frac{\psi}{\theta_{net}})^{k_3}}$$

Equation 4-10

Where; M_R and M_0 are the resilient modulus and initial modulus respectively, θ_{net} = the net mean stress, θ_{ref} = the reference stress state, q_{cyc} = cyclic stress, ψ = matrix suction, and k_1 through k_3 are regression constants that depend on the material at hand. Table 4-1 shows the recommended regression constants for different material.

Table 4-1: Regression Constants (Ng et al., 2013)

Material	AASHTO (2000) classification	Specific gravity	Plastic limit	Liquid limit	Plasticity index	M_0	k_1	k_2	k_3	R^2	Se/Sy
CDT	A-7-6	2.73	29	43	14	8.32	1.00	−0.65	1.01	0.98	0.14
Keuper Marl	A-7-6	2.69	18	37	19	6.32	1.00	−0.65	1.01	0.66	0.60
Gault clay	A-7-5	2.69	25	61	36	0.61	1.00	−0.36	1.31	0.98	0.14
London clay	A-7-5	2.73	23	71	48	0.53	1.00	−0.36	1.31	0.96	0.21

The first term on the right side of the equation denoted the resilient modulus at the reference stress state where the matric suction is equal to zero. The second term quantifies the influence of net mean stress on resilient modulus, showing the increase in stiffness with an increase in confinement. The third term reflects the variation of resilient modulus with cyclic stress, and the fourth term accounts for the effect of matric suction on the resilient modulus. When the matric suction is equal to zero this fourth term reduces to one, and therefore can be applied to saturated soils to find resilient modulus from effective confining pressure and cyclic stress.

4.5 RESILIENT MODULUS PREDICTION EQUATION SUMMARY

Because the resilient modulus has a significant effect directly on the performance of a pavement system, a detailed literature review of the parameter was performed. The goal was to review state-of-the-art equations that were developed to predict resilient modulus of unsaturated soils. The models were either degree of saturation-based or suction-based while the latter varied from completely empirical to more mechanistic constitutive relations. This review identified some of the models with broader application and better prediction capacity and presented for different soil types. It is expected that these models would be implemented in the forthcoming system dynamics model, which would provide the opportunity to better predict the response of pavements with different subgrade material.

CHAPTER 5: SOIL WATER RETENTION EQUATIONS AND HYDRAULIC CONDUCTIVITY MODELS

5.1 INTRODUCTION

Three commonly used predictive Soil Water Retention Curve (SWRC) models are introduced in this section including: Brooks and Corey (BC) Model (Brooks &Corey, 1964), van Genuchten (VG) Model (van Genuchten, 1980), and the Fredlund and Xing (FX) Model (Fredlund & Xing, 1994). A good relationship between the moisture content within a soil and soil suction can be made with direct measurements of the SWRC using different experimental techniques (Lu and Likos 2004). However, these direct measurements are expensive and time consuming. Also, acquiring enough samples from the field to create SWRCs would be expensive given the transportation, lab preparation, and monitoring. Thus, alternative methods were needed to create SWRCs. Numerical approaches, graphical plots, and parameter identification methods were all developed as the alternatives. The three models previously mentioned are examples of these models, which will be discussed.

5.2 SWRC MODELING PARAMETERS

In numerical modeling of SWRC, there are serval different parameters whether they pertain to a certain condition or are an empirical fitting constant. The parameters that pertain to a certain condition such as the soil suction at a specific condition or certain water content include full saturation, residual saturation, and air entry pressure (Lu and Likos 2004). The fitting constants are either empirical or semi-empirical that are selected to capture the general shape of the curve between fixed points; there are two or more within each model.

Some common parameters used within all numerical models are discussed in this section (Lu and Likos 2004). The volumetric water content is expressed as θ and, the saturated water content is represented by θ_S and describes the point where all available pore space within the soil is taken by water. This is usually shown on the curve by the corresponding desorption. The air entry pressure describes the suction on the desorption branch when air first begins entering the largest pores and desaturation begins and is represented by ψ_m. The condition where very little pore water resides in the soil and very large amounts of energy are required to remove it from the matrix is described by the residual water content, θ_r. The degree of saturation is expressed as S, and the effective and residual degrees of saturation are expressed as, S_e and, S_r respectively. An effective degree of saturation can be normalized by the condition ($S = 1$), and if the residual degree of saturation is equal to zero then the effective degree of saturation is equal to the degree of saturation. The commonly used parameter is a dimensionless water content variable, Θ, which is used for modeling purposes. It can be defined by normalizing the volumetric water using Equation 5-1.

$$\boldsymbol{\Theta} = \frac{\boldsymbol{\theta} - \boldsymbol{\theta_r}}{\boldsymbol{\theta_s} - \boldsymbol{\theta_r}}$$

Equation 5-1

5.3 BROOKS AND COREY MODEL

In 1964, Brooks and Corey developed one of the first numerical approaches for modeling the SWRC based on observations from a large array of experiments where water content and suction were directly measured (Brooks and Corey 1964). The equation proposed was a two-part power law relationship that incorporated a “pore size distribution index”, (λ), allowing for different gradations of soil to be modeled. The equation can be both expressed in terms of air entry as in Equation 5-2 or in terms of suction head (h) and air-entry head (h_b) as shown in Equation 5-3.

$$\Theta = S_e = \begin{cases} 1 & \psi < \psi_b \\ (\frac{\psi_b}{\psi})^{\lambda} & \psi \geq \psi_b \end{cases}$$

Equation 5-2

$$\Theta = \boldsymbol{S_e} = \begin{cases} \mathbf{1} & \boldsymbol{h < h_b} \\ (\frac{\boldsymbol{h_b}}{\boldsymbol{h}})^{\lambda} & \boldsymbol{h \geq h_b} \end{cases}$$

Equation 5-3

Figure 5-1 shows the results of data collected for three different soils from a silty sand to a poorly graded sand and was collected using a Tempe cell apparatus. The parameters, gradation index, and porosity for all three different soil types can be seen on the figure (Lu and Likos 2004). Overall, the BC model works best for relatively course grained soils where the drainage occurs of a low and narrow range of suction. Once θ_r is being approached and higher values of suction are present, the model becomes less accurate and less applicable.

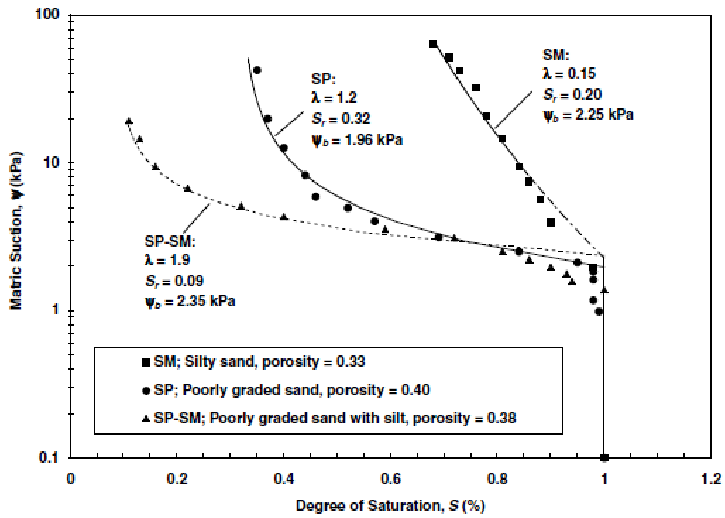

Figure 5-1: Soil-Water Retention Curve Models Using the Brooks and Corey (1964) Model (Lu and Likos 2004)

5.4 VAN GENUCHTEN (VG) MODEL

In 1980, van Genuchten proposed a three-parameter model for the SWRC in a smooth and closed form (van Genuchten, 1980). Smooth transitions at the air-entry pressure and for suction approaching residual condition are more effectively captured, and a wider soil suction range is able to be obtained. The three fitting parameters for the model are represented by α, n, and m. The model is shown in Equation 5-4.

$$\Theta = S_e = \left[\frac{1}{1 + (\alpha\psi)^n}\right]^m$$

Equation 5-4

The n and m fitting parameters correspond to both pore size distribution and the overall symmetry of the characteristic curve. The m parameter is frequently constrained by direct relation to the n parameter, i.e. Equation 5-5 and Equation 5-6, where Equation 5-5 can be used when the residual saturation condition is equal to zero. The α parameter has a unit of inverse pressure.

$$m = 1 - \frac{1}{n}$$

Equation 5-5

$$m = 1 - \frac{1}{2n}$$

Equation 5-6

Figure 5-2 also shows data for three sandy soils where the model parameters can be seen for each corresponding soil on the figures. The model shows an excellent fit to the experimental data over the entire range if the parameters are all fitted independently.

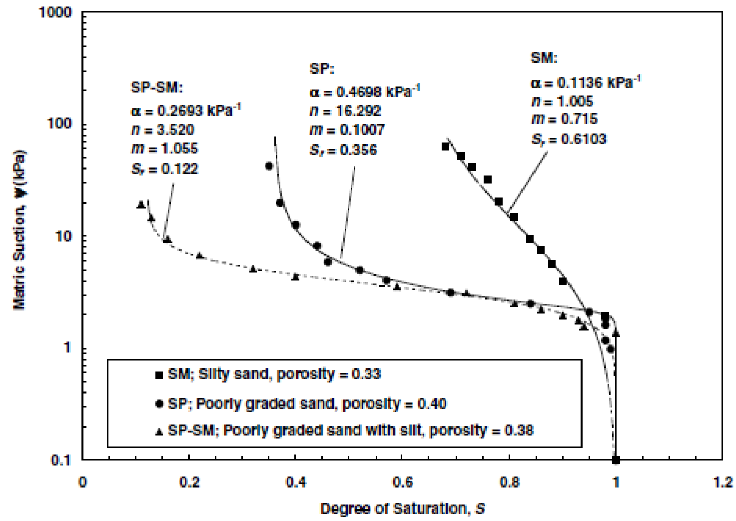

Figure 5-2: Experimental Soil-Water Characteristic Curves Models Using VG (Lu and Likos 2004)

5.5 FREDLUND AND XING (FX) MODEL

In 1994, Fredlund and Xing developed a similar model to VG model in by considering the pore size distribution (Fredlund and Xing, 1994). The model can be seen in Equation 5-7. If the residual water content is assumed to be equal to zero, the model can be written in terms of normalized water content or degree of saturation by dividing both sides of the equation by the volumetric water content.

$$\boldsymbol{\theta} = \boldsymbol{C}(\boldsymbol{\psi})\boldsymbol{\theta}_s \left[\frac{\mathbf{1}}{\ln\left[\boldsymbol{e} + (\frac{\boldsymbol{\psi}}{\boldsymbol{a}})^{\boldsymbol{n}}\right]}\right]^{\boldsymbol{m}}$$

Equation 5-7

The fitting parameters in the equation are a, n, and m. These can be estimated from inflection points located on the measured characteristic curve. Similar to the VG model, the n parameter is related to the pore size distribution and the m parameter is related to the overall symmetry of the characteristic curve. For small values of m, the air-entry value can be used as a. For larger n values, sharper corners near the air-entry value are produced, also more uniform pore distribution are simulated. The m parameter controls the slope of the curve in the higher end of the suction range, where smaller m values result in a steeper slope at higher suction values.

The e parameter is the natural logarithmic constant, and the $C(\psi)$ is a correction factor. This correction forces the model to a suction value of 10^6 kPa at zero water content and can calculated from Equation 5-8.

$$C(\psi) = \left[1 - \frac{\ln(1 + \frac{\psi}{\psi_r})}{\ln(1 + \frac{10^6}{\psi_r})}\right]$$

Equation 5-8

5.6 HYDRAULIC CONDUCTIVITY MODELS

As the degree of saturation decreases from the fully saturated condition, the hydraulic conductivity (or permeability) also decreases. This will become important in the hydraulic water flow analysis. Similar to SWRC, there are several approaches to incorporate the degree of saturation, water content, suction, or head in hydraulic conductivity functions. Among the several available methods Equation 5-9by Gardner (1958) and Equation 5-10 by Brooks and Corey (1964) are simple and commonly used empirical models.

$$k(\psi) = k_s exp(-\alpha\psi)$$

Equation 5-9

where k_s is the saturated hydraulic conductivity and α is indicative of pore size distribution.

$$k(\psi) = \begin{cases} k_s & \psi < \psi_b \\ k_s(\frac{\psi_b}{\psi})^{\eta} & \psi \geq \psi_b \end{cases}$$

Equation 5-10

where η is a fitting parameter.

Also, van Genuchten (1980) and Fredlund et al. (1994) proposed two closed-form solutions developed based on the statistical pore size distribution concept, which are very popular. For example, Equation 5-11 shows the equation by van Genuchten (1980).

$$k(\psi) = \frac{\left[1 - (\alpha\psi)^{n-1}[1 + (\alpha\psi)^n]^{-m}\right]^2}{[1 + (\alpha\psi)^n]^{m/2}}$$

Equation 5-11

5.7 SUMMARY

The soil water characteristic curve describes the relationship between the water content and the suction of a specific soil. Three very commonly used SWRC models were presented and discussed. Although Brooks and Corey model has a smaller number of input parameters but it is less favorable due to its two section equation and less accurate prediction near air entry value. However, the other two models are both effective and would be implemented in the proposed system dynamics model. Further, among different hydraulic conductivity models available in the literature some of the most commonly applied ones were discussed. One model from the simplified approach and one from statistics-based approach will be implemented in the proposed system dynamics simulation.

CHAPTER 6: PAVEMENT MOISTURE PROFILE AND HYDRAULIC MODELING

6.1 INTRODUCTION

Literature was reviewed to investigate how water flows through pavement layers, and how that behavior changes in depth in a given soil. This is a very complex boundary value problem because the way water flows through a given soil depends on several different factors. These factors include pavement structure, the type of soil that the water is flowing through, the hydraulic conductivity of the soil, the current moisture state of the soil, the amount of water that is being introduced to the soil, and the subsurface water level or groundwater level. The other complex piece to this modeling is that depending on these parameters they can all be changing, and the rates at which they change over a given time may be different from one another.

Due to these complexities in this type of system, unsaturated soil models were reviewed based on their simplicity. The boundary conditions of these models will then be attempted to be modified to create more accurate models that may be more applicable to real world scenarios a pavement may experience. Even though an actual pavement system may have saturated soil or partially saturated soil, modeling these systems in a dynamic process becomes more complicated. Also, a review into groundwater recharge and discharge equations was done, to see how the groundwater level changes and how the water moves through a soil system.

6.2 UNSATURATED SOIL FLOW MODELS

There are several different approaches and hydraulic models that are built into pavement design, among them is the Drainage Requirement in Pavement (DRIP) (FHWA, 2002) and Enhanced Integrated Climatic Model (EICM) (Larson and Dempsey, 1997). The DRIP manual from FHWA provides well laid out requirements on pavement subsurface drainage design, several considerations are recommended to be made when creating a hydraulic model for a pavement system. These considerations include: geometric, physical properties of the soil, and the water that enters the pavement system (FHWA, 2002). The geometric considerations primarily focused on the slope of the pavement system. Particularly the resultant, longitudinal, and cross slope of the pavement. Within this information and given the equations, the resultant flow length can be calculated (FHWA, 2002). In terms of the physical properties of the soil, the coefficient of permeability is required and can be calculated based on D_{10} of the soil along with experimental constants. The other physical property of the soil that is required is porosity, which could be difficult to measure without taking a sample of the soil within the pavement system. To include the total water that can enter the pavement, infiltration through cracks, joints, shoulders, and side ditches were incorporated, along with meltwater. Groundwater variation was mentioned as an important factor, but no equations were presented to estimate the effect of this variation. To estimate infiltration, two different methods are recommended, the crack infiltration method and the infiltration ratio method. These equations are based on infiltration rates, both pavement and soil, geometric properties of the cracks, rainfall rates, and experimental constants (FHWA, 2002).

A challenge with the DRIP model is that it is focused on the flow of water into the pavement, through cracks and different forms. This is valuable when one desires to recognize the factors that should be built into the proposed model. However, it is also important to know how the groundwater moves within the soil. That is why additional models were reviewed, particularly models that deal with unsaturated soil flow. EICM, which is also used in the Pavement ME system, relies on a 1D finite difference modelling of moisture flow through pavement. A limitation though for EICM is that it does use a constant ground water table boundary condition. Also, EICM relies on 1-D modeling which does not accurately replicate the 3-D flow problem.

In terms of soil moisture movement models, some of these are empirical in nature such as: Kostiakov's Equation, Horton's Equation and the SCS equation (Ravi and Williams, 1998). These models are a good basis, but they are empirical. The other two models that were reviewed are the Green-Ampt Model and the Richards' Equation. The Richards' Equation is one of the most commonly used method and the most accurate one. The problem with this equation is that it involves many differential equations and boundary conditions, which make it very complex. Therefore, the Green-Ampt model was reviewed, which is also an accurate model, but makes some assumptions that results in a simpler equation to be modeled. More details on this model are provided next.

The Green-Ampt Model was developed in 1911, by Green and Ampt. The model was one of the first models commonly used to describe how water moves through soil based on the simplicity of the model and the accurate results that it yielded (Ravi and Williams, 1998). The basic parameters that the model includes is the water pressure from infiltrating water from above the soil, the hydraulic conductivity of the soil, the volumetric water content of the soil, and time. Knowing the physical properties of the soil and properties of the infiltration rate on the soil, the wetting front level, which can be related to the groundwater level, can be estimated for any given time. To find the critical time, which is defined as the duration of flooding needed to cause a completely saturated base course condition, can be estimated with an explicit form of the Green-Ampt equation shown below in Equation 6-1 (FHWA, 2019).

$$\boldsymbol{t = \frac{\theta_s - \theta_i}{k}[L_f - (h_L - \varphi_f)\ln\left(\frac{h_L + L_f - \varphi_f}{h_L - \varphi_f}\right)]}$$

Equation 6-1

Where:
Θ_s = volumetric moisture content at saturation
Θ_i = initial volumetric moisture content
L_f = thickness of (HMA + base course), m
Φ_f = suction, m
h_L = depth of ponded water, m
t = time to infiltrate, seconds
k = permeability, m/s

In order to account for the flow of water through the HMA layer above the base course material, an effective permeability is estimated (FHWA, 2019). This estimation can be made with Equation 6-2.

$$\boldsymbol{k_{effective}} = \frac{\boldsymbol{h_{HMA}} + \boldsymbol{h_{Base}}}{\frac{\boldsymbol{h_{HMA}}}{\boldsymbol{k_{HMA}}} + \frac{\boldsymbol{h_{Base}}}{\boldsymbol{k_{Base}}}}$$

Equation 6-2

Where:
$k_{effective}$ = effective permeability in m/s
h_{HMA} = thickness of HMA, m
h_{Base} = thickness of base course, m
k_{HMA}= permeability of HMA, m/s
k_{Base} = permeability of base course, m/s

Through iterations of analyzing the pavement system with the Green-Ampt model, several conclusions could be made (FHWA, 2019). The first is that the critical time is significantly affected by the HMA layer permeability, and in almost all cases this critical time was within six hours and frequently within two hours. Another conclusion is that cracks within the pavement system, especially for thin HMA surfaces, can greatly increase the amount of water that enters into the pavement system. The most vulnerable time for HMA pavements was observed to be right after construction due to the voids in the system being relatively high. Finally, climatic considerations and locations are important factors, as temperature and distance to nearby water sources have the possibility to significantly affect the HMA system (FHWA, 2019).

However, this model has a few issues based on the assumptions made. One of these issues is that the model assumes completely unsaturated soil and a water table depth at some level below the soil. This is an unfavorable assumption, because in a case where a pavement system becomes flooded, the soil would be completely saturated. Another issue this model has is that an estimate is made with an effective permeability to explain how water flows through the pavement material; however, this isn't necessarily the most accurate estimation. The final problem with the model is that it assumes one dimensional flow straight down from the surface into the pavement, while in reality, water may be flowing in multiple directions (Ravi and Williams, 1998).

There are different hydraulic analysis software that have been used in the literature such as MnDrain (FHWA, 2019) or VADOSE/W (GeoSlope, 2019). MnDrain is a software developed by the Minnesota Department of Transportation for dealing with unsaturated flow within a pavement system. Within this software the Brooks and Corey model is used to relate hydraulic conductivity and pressure, or moisture content (FHWA, 2019). The equations used in this software can be seen below in Equation 6-3 and Equation 6-4. VADOSE/W is also part of Geo-Studio package that deals with flow in unsaturated soil zones above the water table.

$$K = K_{sat}\left[\frac{\theta - \theta_{res}}{\theta_{sat} - \theta_{res}}\right]^{\lambda 2}$$

Equation 6-3

$$\theta = \begin{cases} 0.0001\,(\psi - \psi_b) + \theta_{sat} & \psi < \psi_b \\ (\theta_{sat} - \theta_{res})\left(\frac{\psi}{\psi_b}\right)^{-\lambda_1} + \theta_{sat} & \psi \geq \psi_b \end{cases}$$

Equation 6-4

Where:
K = Hydraulic conductivity
K_{sat} = Saturated hydraulic conductivity
λ_1 and λ_2 = material constants

Among other developed models for flow through pavement systems is the FLODEF package (Long et al. 2006). In this model the moisture flow is simulated using a 2D Finite Element numerical model based on Lytton's approach for moisture flow-soil deformation response. The method used a simplified Mitchell and Avalle's (1984) procedure for solving a moisture diffusion problem. In this method, the suction-permeability relation for unsaturated soil is based on the relationship proposed by Laliberte et al. (1966), which is similar in form to the one proposed by Brooks and Corey (1964). In a similar study, Espinoza and Bourdeau (1992) developed a computer program PURDRAIN to model the water infiltration with the pavement systems, which was based on theories of water flow in unsaturated soils.

6.3 GROUNDWATER RECHARGE MODELS

The next type of models are groundwater recharge and discharge models. There are several different types of these models, but similarly to the unsaturated flow models, many of them are empirical or too complex (Freeze 1969, Pathak 2014). Among these a model by Freeze (1965) is reviewed here. The model assumes one dimensional vertical flow in an unsaturated system. This model can be seen in Figure 6-1(Freeze, 1969). As seen, the model incorporates the pressure heads, volumetric water content of the soil, depth, and other physical properties. Also, the model incorporates both infiltration and evaporation into the system. The issue with the model is that it most likely will not be able to be directly incorporated into the proposed model. This is for the same reason as the Green-Ampt Model, because both are solved for a wetting front moving down in an unsaturated zone, which is different than the recession of groundwater level. However, the proposed differential equation forms can be adapted for the given boundary value problem in the proposed project, in order to understand how the water recedes after flooding and to estimate the moisture profile in depth.

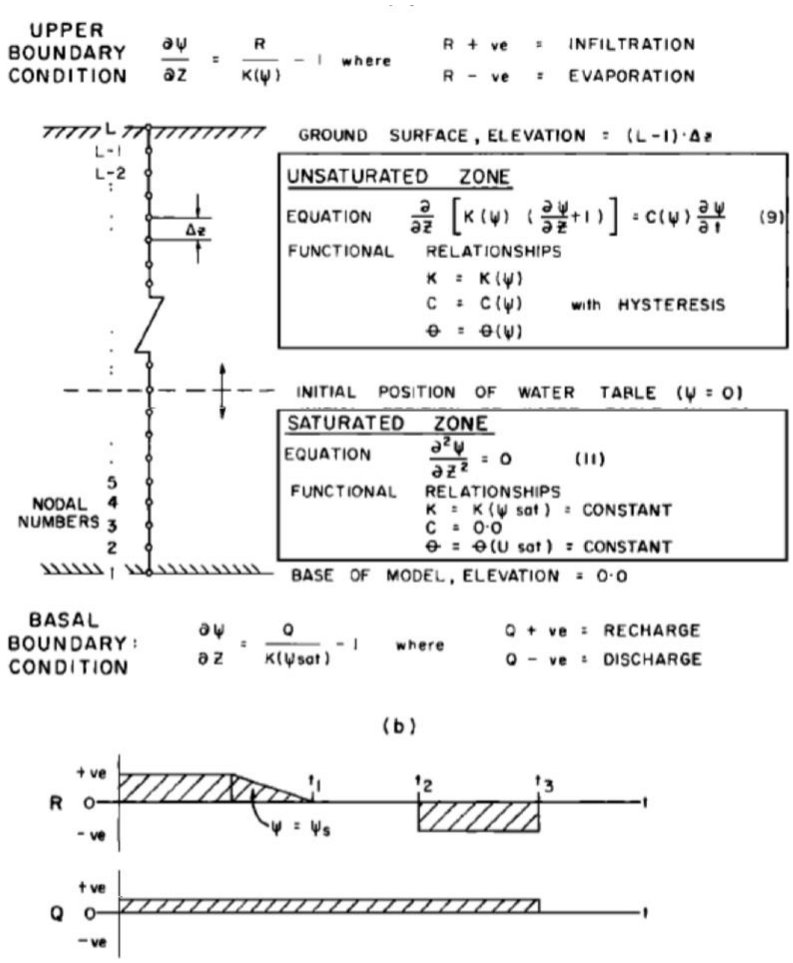

Figure 6-1: Groundwater Recharge Model (Freeze 1969)

6.4 PAVEMENT MOISTURE PROFILE AND HYDRAULIC MODELING SUMMARY

A key factor in accurate assessment of pavement performance during and after inundation is the understanding of how water flows within the pavement system. In an event of flooding the soil is first saturated, but as water dissipates through the soil, the soil then becomes unsaturated and then unsaturated. Thus, a correct model should incorporate a dynamic flow system with a suction (or degree of saturation) – dependent permeability. Among the different classes of models presented in the review, the Green-Ampt model or the ground water recharge models can be adjusted for the given hydrologic scenarios of interest. Starting with the core differential equations in these methods and solving for the boundary value problem, a simple semi-analytical procedure will be included in the proposed system dynamics model.

CHAPTER 7: LITERATURE REVIEW SUMMARY AND CONCLUSIONS

7.1 SUMMARY

Literature was reviewed in order to prepare for the next steps within this research project. This included: why load restrictions were needed to understand the importance of this research, basic pavement design components to get familiar with how the systems perform, different parameters that affect the performance of the pavement to see how a change in moisture affects each parameter, and different models that look at how water flows through soil to understand the behavior of water within the pavement system after and during a flooding event.

Key findings from the literature review resulted in understanding what parameters have a significant impact on the performance of the pavement system. Predictive equations for each of these parameters were then identified, which can ultimately be used in the system dynamics model to help predict the overall performance of the pavement system due to a storm event (or other events that result in excessive moisture within pavement system). For resilient modulus, the equations were based on how they were used in previous implementations, these were presented in CHAPTER 4 . For the soil water retention curves, the two most commonly used models were chosen to incorporate into the system dynamics model, along with the two of the hydraulic conductivity models. These models were presented in CHAPTER 5 . Incorporating these reviewed models in a system dynamics approach, will help to have a better understanding of how pavement performance is affected by a change in moisture within the system. CHAPTER 6 presented alternative approaches on simulating the water flow in pavement layers and subgrade soils, which is key to estimation of the moisture profile in depth.

7.2 LITERATURE NOT REVIEWED

As already mentioned, multi-directional flow was not fully investigated. Another area of literature that was not reviewed was climatic predictions. These were not within the scope of the project, and only public domain climatic forecast data will be used in the current work. However, the model can be adapted and cross-linked in future for climatic forecasting.

7.3 FUTURE WORK

In future, the literature will be continuously reviewed to update the models based on the state-of-the-art and practice. Further investigation into more accurate hydraulic flow models should be made, specially solving the differential equations for the boundary value problem in hand. This literature review provided the basic understanding of how moisture flows in a soil system and the assumptions that are made in a hydraulic model. The basic parameters to incorporate into the model are now available along with relationships and equations to estimate them. If a relationship or equation needs to be adjusted in the future, the model can always be adjusted and revisited to account for this change. An on-going task (Task 3) of this project will develop and discuss a system dynamics framework with selected models and equations. This framework will be first presented to the project technical advisory panel (TAP) to obtain their feedback. Once the framework is finalized, its implementation in system dynamics software, VENSIM will ensue.

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APPENDIX

Summary of Resilient Modulus Equations

TYPES OF SOIL USED = PM= Pulverized mudstone; DT= Decomposed Tuff; SCL= Silty Clay Loam								
Relationship	**Assumptions**	**Equation**	**Group (Type of Equation)**	**Recommended Soil Type**	**Where Implemented**	**When it was used**	**Supported by Others / Notes**	**Soil Suction Range (kPa)**
MEPDG	Used to calibrate the optimum resilient modulus values; also developed with degree of saturation where SWCC describes the relationship	Eq. (3) is used in MEPDG to calibrate the M_{ROPT} values considering the influence of seasonal moisture content fluctuations $\log\left(\frac{M_R}{M_{ROPT}}\right) = a + \frac{b-a}{1+\exp\left[\ln\left(-\frac{b}{a}\right) + k_m(S - S_{OPT})\right]}$ (3)	(A) Emperical	-	-	2004	One of the main equations used with a variety of soil types and scenerios; R^2 = 0.88 when compared	-
Swangsuriya et al.	Four fine-grained soils (2 A-4 and 2 A-7-6)	$M_R/M_{RSAT} = -5.61 + 4.54\log(\psi)$	(A) Emperical	Fine-Grained	Minnesota	2009	R^2 = 0.68 tends to under predict most data; sensitive to saturated resilient modulus; small differences contribute to significant variations in predicted resilient modulus	0-10,000
Yang et al.	Two fine-grained subgrade soils (A-7-5 and A-7-6)	$M_R = k_1(\sigma_d + \chi\psi)^{k_2}$	(B) Constitutive Models	R^2>0.9; predicts different non-linear Mr-ω at various levels of shearing stress for PM and SCL; Extend the independent stress state variable approach and reasonably takes into account of soil suction	Taiwan, China	2005	R^2=0.56; reasonably captures increase in Mr with ω; most suitable to predict Mr-ω correlations for subgrade soils that exhibit hardening behavior (with respect to applied shearing stresses)	0-10,000
Liang et al.	Two fine-grained subgrades (A-4 and A-6); also validated using 8 sets of experimental data on fine-grained soils from lit.	$M_R = k_1 p_a\left(\frac{\theta_b + \chi\psi}{p_a}\right)^{k_2}\left(1 + \frac{\tau_{oct}}{p_a}\right)^{k_3}$	(B) Constitutive Models	Fine-Grained	-	2008	R^2=0.95; predicted behavior of Mr with respect to σd for PM and SCL are not consistent	150-380
Khoury et al.	Several subgrade soils (range from A-4 to A-7)	$M_R = k_1 p_a\left(\frac{\theta_b}{p_a}\right)^{k_2}\left(k_4 + \frac{\tau_{oct}}{p_a}\right)^{k_3} + \alpha_1\psi^{\beta_1} = M_{RSAT} + \alpha_1\psi^{\beta_1}$	(C) Consecutive models extending the independent stress state variable approach	Subgrade Soils (Range from A-4 to A-7)	Oklahoma	2009	R^2>0.9; predicts variation of Mr regardless of influence of applied shearing stress; does not consider influence of applied stress on the Mr-ω relationships	0-6,000
Ng et al.	Subgrade soil	$M_R = M_0\left(\frac{p}{p_r}\right)^{k_1}\left(1 + \frac{q_{cyc}}{p_r}\right)^{k_2}\left(1 + \frac{\psi}{p}\right)^{k_3}$	(C) Consecutive models extending the independent stress state variable approach	Subgrade Soils	Hong Kong, China	2013	R^2>0.9; predicts different non-linear Mr-ω at various levels of shearing stress for PM and SCL; Extend the independent stress state variable approach and reasonably takes into account of soil suction and provide reliable predictions within boundary effect and transition zones	0-250
Johnson et al.	-	$M_R = 1.35 \times 10^6 \times (101.36 - \psi)^{2.36}(J_1)^{3.25}(\gamma_d)^{3.06}$	(A) Emperical	Sandy Soils	-	1986	-	-
Parreira and Gonçalves	A-7-6 Soils	$M_R = 14.10\sigma_d^{0.782}\psi^{0.076}$	(A) Emperical	Lateritic Soil	Brazil	2000	-	0-87,500 kPa
Ceratti et al.	A-7-6 Soils	$M_R = 142 + 16.9\psi$	(A) Emperical	Lateritic Soil	Brazil	2004	-	0-14 kPa

Doucet and Dore	Also developed for several "partially crushed materials"	$M_R = 10600\theta_b - 8700\psi + 57000$	(A) Emperical	Crushed Granular Materials	Quebec, Canada	2004	-	
Swangsuriya et al.	Four fine-grained soils (2 A-4 and 2 A-7-6)	$M_R/M_{ROPT} = -0.24 + 0.25\log(\psi)$	(A) Emperical	Fine-Grained	Minnesota	2009	-	0-10,000 kPa
Ba et al.	Dervied for four unbound granular base materials	$M_R/M_{ROPT} = 0.385 + 0.267\log(\psi)$	(A) Emperical	Granular	Senegal	2013	-	0-100 kPa
Moossazadeh and Witczak	Relates applied stress using model parameters	$M_R = k_1\left(\frac{\sigma_d}{p_a}\right)^{k_2}$	(B) Constitutive Models	-	-	1981	Most commonly used constitutive models	-
Uzan	Relates applied stress using model parameters	$M_R = k_1 p_a\left(\frac{\theta_b}{p_a}\right)^{k_2}\left(\frac{\tau_{oct}}{p_a}\right)^{k_3}$	(B) Constitutive Models	-	-	1985	Most commonly used constitutive models	-
Loach	-	$M_R = \frac{\sigma_d}{k_1}\left[\frac{c\sigma_c + \psi}{\sigma_d}\right]^{k_2}$	(B) Constitutive Models	Fine Grained Soils	United Kingdom	1987	-	0-100 kPa
Jin et al.	granular base materials	$\Delta M_R = K_1 K_2 \theta_b^{K_2-1}(\Delta\theta_{bT} + \Delta\theta_{b\psi})$	(B) Constitutive Models	Granular	Rhode Island	1994	-	-
Lytton	granular base materials	$\Delta M_R = K_1 K_2 \theta_b^{K_2-1}(\Delta\theta_{bT} + \Delta\theta_{b\psi})$	(B) Constitutive Models	Granular	-	1995	-	-
Gu et al.	Verification of eq. 14	$M_R = k_1 p_a\left(\frac{\theta_b - 3f\theta\psi}{p_a}\right)^{k_2}\left(\frac{\tau_{oct}}{p_a}\right)^{k_3}$	(B) Constitutive Models	Granular	Texas	2014	Derived off of nine granular base materials from Texas	-
Heath et al. 2004	Based off of a typical base material located in California	$M_R = k_1 p_a\left[\frac{(\theta_b/3) - u_a + \chi\psi}{p_a}\right]^{k_2}\left(\frac{\sigma_d}{p_a}\right)^{k_3}$	(B) Constitutive Models	Most likely granular "base material"	California	2004	-	-
Oh et al.	derived for both base and subgrade materials	$M_R = k_1 p_a\left(\frac{\theta_b + 3k_4\psi\theta}{p_a}\right)^{k_2}\left(\frac{\tau_{oct}}{p_a} + 1\right)^{k_3}$	(B) Constitutive Models	Granular	Florida	2012	-	-
Sahin et al.	Base materials	$M_R = k_1 p_a\left[\frac{\theta_b - 3f\theta(\psi_0 + \beta\frac{\theta_b}{3} + \alpha\tau_{oct})}{p_a}\right]^{k_2}\left(\frac{\tau_{oct}}{p_a} + 1\right)^{k_3}$	(B) Constitutive Models	Granular	-	2013	-	-

Fredlund et al.	-	$\log M_R = c_{ld} - m_{ld}(\sigma_d)$	(C) Consecutive models extending the independent stress state variable approach	Glacial Till	Saskatchewan, Canada	1977	-	0-1000 kPa
Oloo and Fredlund 1998	Coarse-grained soils (eq.22); Fine-grained soils (eq.23 & 24)	$M_R = k\theta_b^{m_b} + k_s\psi$ $M_R = k_2 - k_3(k_1 - \theta_b) + k_s\psi$ when $k_1 > \theta_b$ $M_R = k_2 + k_4(\theta_b - k_1) + k_s\psi$ when $k_1 < \theta_b$	(C) Consecutive models extending the independent stress state variable approach	Coarse-Grained and Fine-grained	-	1998	-	-
Gupta et al. 2007	Two a-4 soils and two A-7-6 soils	$M_R = k_1 p_a \left(\frac{\theta_b - 3k_4}{p_a}\right)^{k_2} \left(k_5 + \frac{\tau_{oct}}{p_a}\right)^{k_3} + \alpha_1 \psi^{\beta_1}$ $M_R = k_1 p_a \left(\frac{\theta_b}{p_a}\right)^{k_2} \left(1 + \frac{\tau_{oct}}{p_a}\right)^{k_3} + k_{us} p_a \Theta^{\kappa} \psi$	(C) Consecutive models extending the independent stress state variable approach	Fine-Grained	Minnesota	2007	-	10-10000 kPa
Caicedo et al.	dervied from three nonstandard base materials	$M_R = k_1 p_a \left(1 + k_2 \frac{\sigma_d}{p_a}\right) \left(\frac{\psi}{p_a}\right)^{k_3} \frac{f(e)}{f(0.33)}$	(C) Consecutive models extending the independent stress state variable approach	Granular	Andes Cordillera, Colombia	2009	-	0-200 kPa
Khoury et al.	Derived from manufacured soil	$M_R = \left[k_1 p_a \left(\frac{\theta_b}{p_a}\right)^{k_2} \left(1 + \frac{\tau_{oct}}{p_a}\right)^{k_3} + (\psi - \psi_0) \times \left(\frac{\theta_d}{\theta_s}\right)^{\left(\frac{1}{n}\right)}\right] \times \left(\frac{\theta_d}{\theta_w}\right)$	(C) Consecutive models extending the independent stress state variable approach	Silty Soil	-	2011	Hysteresis behavior in Mr	0-100 kPa
Cary and Zapata	Further verified by Salour et al. 2014 in Sweden; Two sandy subgrade soils (A-4 and A-2-4 with soil suction range 0-450kPa)	$M_R = k_1 P_a \left(\frac{\theta_{net} - 3\Delta u_{w\text{-}sat}}{p_a}\right)^{k_2} \left(\frac{\tau_{oct}}{p_a} + 1\right)^{k_3} \left(\frac{\psi_0 - \Delta\psi}{p_a} + 1\right)^{k_4}$	(C) Consecutive models extending the independent stress state variable approach	Granular Soil and Clayey Soil (further research needed for fine cohesive soils)	Arizona	2011	Considers dynamic loading and resulting change in pore water pressure; model proposed is modification of the MEPDG that accommodates changes in matrix suction and effects of drainage conditions; triaxial testing	0-250 kPa
Azam et al.	derived from recycled unbound materials	$M_R = k\left(\frac{\sigma_m}{p_a}\right)^{k_1} \left(\frac{\tau_{oct}}{\tau_{ref}}\right)^{k_2} \left(\frac{\psi}{p_a}\right)^{k_3} \left[\frac{\mathrm{DDR}(1 - k_4 \mathrm{RCM}/100)}{100}\right]^{k_5}$	(C) Consecutive models extending the independent stress state variable approach	Granular	Australia	2013	-	0-10 kPa
Han and Vanapalli	Derived from experiments from 11 compacted fine-grained subgrade soils	$M_R = k\left(\frac{\sigma_m}{p_a}\right)^{k_1} \left(\frac{\tau_{oct}}{\tau_{ref}}\right)^{k_2} \left(\frac{\psi}{p_a}\right)^{k_3} \left[\frac{\mathrm{DDR}(1 - k_4 \mathrm{RCM}/100)}{100}\right]^{k_5}$ $\frac{M_R - M_{RSAT}}{M_{ROPT} - M_{RSAT}} = \frac{\psi}{\psi_{OPT}} \left(\frac{S}{S_{OPT}}\right)^{\xi}$	(C) Consecutive models extending the independent stress state variable approach	Fine-Grained	-	2015	Has large list of protocols followed during testing (listed in paper)	-