APPENDIX A

BASIC FORMULATION OF THE

COMPUTER PREDICTION MODEL

BASIC FORMULATION OF THE COMPUTERIZED PREDICTION MODEL

This appendix presents the basic formulation for the traffic noise prediction scheme utilized by the computer program. The first subsection describes the theory supporting the prediction of the overall A-weighted energy mean sound level of the traffic flow on a finite roadway segment. Distance attenuation is assumed to be in excess of classical cylindrical spreading from a line source.

Since the STAMINA 2.0 program is an evolutionary development of the TSC MOD-04 and STAMINA 1.0 programs, the notation used in this appendix is consistent with the latter models (1, 2).

Section A.10 of this appendix presents a discussion of the formulation of the barrier effectiveness/cost ratios as computed in the OPTIMA program.

A.1 Energy Mean Sound Level

The geometric relationship between the roadway segment and the receiver is illustrated in Figure A-1. Considering the traffic flow to be represented by a uniform source strength, λ distributed along the roadway segment, the acoustic intensity ratio at the receiver is expressed as:

$$I/I_o = \lambda 10^{\bar{L}_o/10} (D_o/D)^{\gamma+2} \; 10^{-\varepsilon/10} \int_{\bar{x}_1}^{\bar{x}_2} \frac{dx}{[1+(x/D)^2]^{\gamma+2}} \qquad \text{(A-1)}$$

where $\bar{L}_o$ is the reference sound level at a distance D_o from the roadway segment

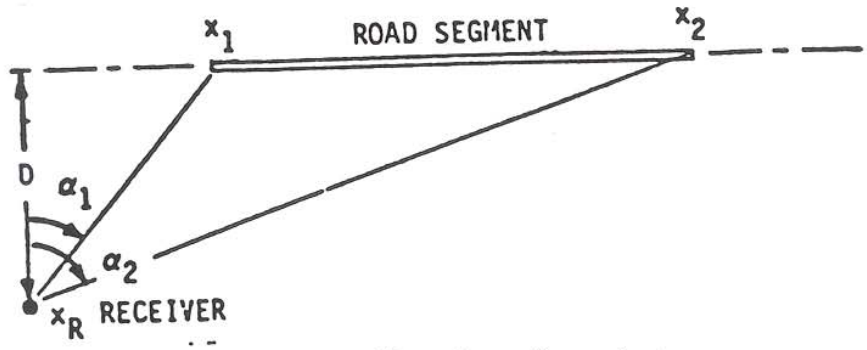
D is the distance from the roadway segment to the receiver

λ is the source concentration (See Equation (A-6)

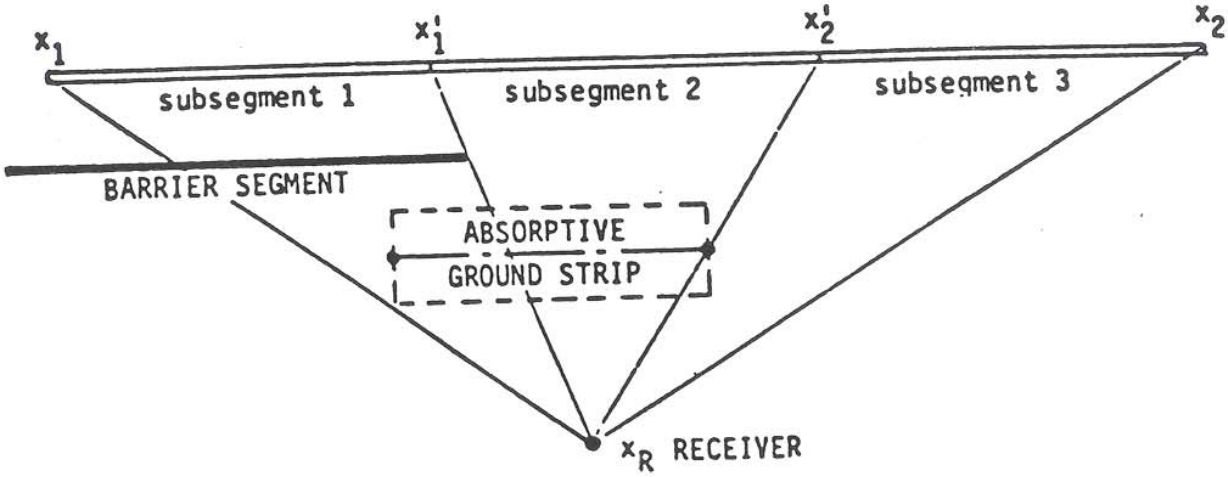
ε is an excess attenuation parameter independent of distance

γ is an excess attenuation parameter proportional to distance from the source

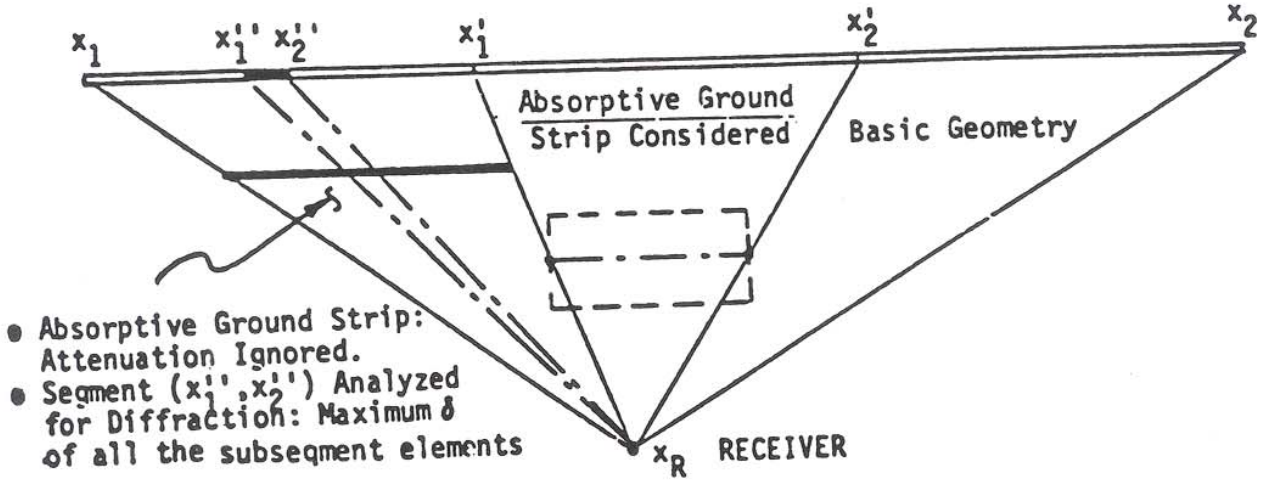
$I_o = (p^2 ref)/pc$, the reference intensity



(a) Basic Roadway/Receiver Geometry



(b) Subdivision of Roadway Segments into Elements



(c) Computer View of Roadway/Receiver Geometry

FIGURE A-1. BASIC SOURCE/RECEIVER GEOMETRY

The relationship indicated in Equation (A-1) assumes that all sources on the roadway segment are identical nondirectional sources. Distance attenuation is assumed to follow an attenuation rate in excess of inverse square spreading as indicated by the parameter γ.

$$x = D \tan(\alpha) \qquad \text{(A-2)}$$

The result obtained is:

$$I/I_0 = \lambda D_0 \, 10^{(\bar{L}_0 - \varepsilon)/10} \, (D_0/D)^{\gamma+1} \, \Psi_\gamma(\alpha_2, \alpha_1) \qquad \text{(A-3)}$$

where $$\Psi_\gamma(\alpha_2, \alpha_1) = \int_{\alpha_1}^{\alpha_2} \cos^\gamma(\alpha)\, d\alpha$$

It will be noted that the function $\psi_\gamma(\alpha 2, \alpha 1)$ is identical to the "Finite length roadway adjustment factor" used in the FHWA Level 1 formulation (2).

From Equation (A-3) the energy mean sound level is

$$L_e = 10 \log (I/I_0) = \bar{L}_0 - \varepsilon + 10 \log (\lambda D_0 (D_0/D)\, I^*) \qquad \text{(A-4)}$$

where $I^* = (D_0/D)^\gamma \, \Psi_\gamma(\alpha_2, \alpha_1)$

One of the prediction tasks of the FHWA Level 2 program is to estimate the percentile sound levels generated by the traffic flow.

To accomplish this, Kurze's traffic flow noise theory is used (3), Kurze's theory assumes a uniform distribution of identical point sources along the finite roadway segment.

To estimate the percentile sound levels, it is required to calculate statistical moments (called cumulants) for constant speed road traffic: By normalizing the acoustic intensity at the receiver by its energy mean value (given by Equation A-4) and performing some algebra (See Reference 3) one obtains the following expression for the nth order cumulant:

$$\kappa_n = \frac{(N/(\bar{x}_1-\bar{x}_2))\phi_\gamma^n(\alpha_1,\alpha_2)}{\lambda^n D^{n-1}[\Psi_\gamma(\alpha_1,\alpha_2)]^n} \qquad \text{(A-5)}$$

where

N is the number of vehicles occupying the roadway segment

$$\phi_\gamma^n(\alpha_2, \alpha_1) = \int_{\alpha_1}^{\alpha_2} [\cos(\alpha)]^{\eta(\gamma+2)-2} d\alpha$$

λ, D, γ are defined in Equation (A-1)

$\Psi_\gamma(\alpha 2, \alpha 1)$ is defined in Equation (A-3)

$\bar{x}_1, \bar{x}_2, \alpha_1, \alpha_2$ are defined in Figure A-1

By normalizing the acoustic intensity at the receiver by its energy mean sound level, the first order cumulant must be identically equal to unity. From this requirement and the assumption of a uniform distribution of noise sources along the roadway segment, the source concentration, λ, is defined as

$$\lambda \equiv N/(\bar{x}_2-\bar{x}_1) = Q/V, \text{ vehicles per unit length} \qquad \text{(A-6)}$$

where Q is the traffic flow rate

V is the traffic speed

Using a first order approximation, the standard deviation of the fluctuating sound level at the receiver is estimated by (3)

$$\sigma_L = (10/\ln(10)) \sqrt{\ln(1+\kappa_2)} \qquad \text{(A-7)}$$

where $\kappa_2 = \dfrac{\phi_\gamma^2(\alpha_2, \alpha_1)}{\lambda D[\Psi_\gamma(\alpha_2, \alpha_1)]^2}$ is the second order cumulant

The relationship of Equation (A-7) is exact for a Gaussian or normal distribution of sound levels.

By setting the excess distance attenuation factor, γ to zero the model described by the TSC MOD-04 code is obtained.

The results of this section apply to a single vehicle type operating on a finite roadway segment.

A.2 Coded Formulation

The coding changes required to incorporate the theory of Sec. A.1 into the TSC MOD-04 program to create the STAMINA program are related to the numerical integration of the functions $\Psi_\gamma(\alpha 2, \alpha 1)$ and $\phi_\gamma^2(\alpha 2, \alpha 1)$. These changes are confined to SUBROUTINE GEOMETRY.

Using a subscript "i" to denote a roadway segment and a subscript "j" to denote a traffic flow condition (vehicle type and speed condition) on the roadway, the energy mean sound level at the receiver is formatted as

$$L_e = 100 + 10 \log (I^*) \qquad \text{(A-8)}$$

The acoustic intensity at the receiver is*

$$I^* = \sum_{i=1}^{NR} \{[(D_o/D_i)^{\gamma_i} \Psi_{\gamma_i}(\alpha_{2i},\alpha_{1i}) /D_i]\{ \sum_{j=1}^{NQS} \lambda_{ij}\, 10^{(\bar{L}_{oij}-66-\varepsilon_{ij})/10} \times e^{\frac{1}{2}(\bar{\sigma}_{ij}/4.35)^2}\} \quad \text{(A-9)}$$

and the second moment of the acoustic intensity is

$$\kappa_2 = \kappa_2^*/(I^*)^2 \quad \text{(A-10)}$$

$$\kappa_2^* = \sum_{i=1}^{NR}\left\{[(D_o/D_i)^{2\gamma_i}\phi_{\gamma_i}(\alpha_{2i},\alpha_{1i}) /D_i^3]\left\{\sum_{j=1}^{NQS} \lambda_{ij}\, 10^{2(\bar{L}_{oij}-66-\varepsilon_{ij})/10} e^{\frac{1}{2}(2\bar{\sigma}_{ij}/4.35)^2}\right\}\right\} \quad \text{(A-11}$$

$$\Psi_{\gamma_i}(\alpha_{2i}, \alpha_{1i}) = \int_{\alpha_{1i}}^{\alpha_{2i}} [\cos(\alpha)]^{\gamma_i}\, d\alpha \qquad \phi_{\gamma_i}(\alpha_{2i}, \alpha_{1i}) = \int_{\alpha_{1i}}^{\alpha_{2i}} [\cos(\alpha)]^{2(\gamma_i+1)}\, d\alpha$$

These results assume that the distribution of sound levels from a vehicle condition "j" (i.e. vehicle type and speed) are normally distributed. The above results apply both to the overall A-weighted sound level and the octave band sound level predictions conducted by the computer program. As indicated in Equations (A-9) and (A-10), the attenuation parameter, y_i, applies for each roadway-receiver combination.

The integrations indicated following Equation (A-11) are calculated numerically by the program.

If the attenuation parameter, γ_i, is set to zero in the above results, the formulation is identical to the theory of the TSC MOD=04 prediction model.

NOTE: For D_o = 50 feet; $D^2 = (50)^2 = (100/2)^2 = 10^4 . 10^{-6/10}$
$= 10^{10} . 10^{-66/10}$

(i.e., Equations (A-9) and (A-10) are for length expressed in feet)

The estimation of percentile sound levels from the roadway is based upon the assumption that the distribution of sound level during the hour is Gaussian or Normal. Based upon the estimate of the overall A-weighted sound level, L_e (Equation (A-8)), and the standard deviation, σ_L (Equation (A-7)), the percentile sound levels are:

$$L_{50} = L_e - \sigma_L^2/8.7 \quad , \text{dB} \qquad \text{(A-12)}$$

$$L_{10} = L_{50} + 1.25\sigma_L \quad , \text{dB} \qquad \text{(A-13)}$$

$$L_{90} = L_{50} - 1.25\sigma_L \quad . \text{dB} \qquad \text{(A-14)}$$

A.3 Spectral Calculations

Although the predecessors to STAMINA 2.0 1, 2 perform spectral calculations by octave band, STAMINA 2.0 performs calculations for the overall A-weighted sound level only. However, throughout most of the code, the statements that performed the spectral calculations have been left in, but commented so that they are not executed.

For propagation over barriers, ground strips and through the atmosphere, excess attenuation is calculated at 500 Hz, and that attenuation is attributed to the overall A-weighted sound level.

A.4 Attenuation of Sound Levels

The traffic noise prediction code provides for the consideration of the attenuation of sound levels from the source to the receiver due to the following physical factors:

* Distance between source and receiver
* Barriers between source and receiver
* Trees and shrubbery between source and receiver
* Atmospheric absorption
* Reflection of sound to the receiver

  (negative attenuation)

The basic attenuation included in the acoustic model is an inverse square law spreading of sound intensity (i.e., 3 dB per distance doubling) that is frequency independent. All other forms of excess attenuation considered by the prediction code consider both frequency and distance effects in calculating attenuation as may be appropriate to the models utilized.

By plotting the predicted values of equivalent sound level, LE(H), and the statistical levels ($L_{90'}$, $L_{50'}$, and L_{10}) versus distance, the user will note slight differences in distance attenuation rates for the different descriptors. The reason for this is that both sound level and the composite value of standard deviation decrease with distance at different rates (see Equation (A-9)).

The statistical sound level descriptors are functions of both the equivalent sound level and the standard deviation of the sound level – hence, one would expect to observe differences.

A.5 Atmospheric Absorption

The traffic noise prediction code utilizes an empirical formula for the attenuation of sound resulting from atmospheric absorption. This formula is dependent upon frequency and distance between the source and the receiver and is specialized for ambient temperatures around 68 F and relative humidity in the range of 50 to 70 percent (4).

The empirical formula utilized is

$$A = 5.4 \cdot 10^{-4} \; r \quad dB \qquad (A\text{-}16)$$

Where

r is the source-receiver distance in feet

Attenuation of sound for atmospheric absorption is accomplished in Subroutine Geomry. For the overall A-weighted sound level prediction, the value used is that for 500 Hz.

The distance used by the prediction code in calculating atmospheric absorption is the distance from the receiver to the nearest point on the roadway segment or sub-segment being analyzed.

A.6 Diffraction

Diffraction of sound is caused by obstacles in the direct or reflected propagation paths from the roadway to the receiver. Such obstacles can be artificial barriers, earth berms, hills, buildings, etc. For the calculation of attenuation by diffraction, the obstacle can be modeled by a rigid, impervious screen oriented perpendicular to the ground plane so that sound is diffracted over the top edge of the screen exclusively. The shape of hills and the thickness of barriers are neglected because of the lack of available knowledge. The sound absorption and transmission properties of barriers are not considered because they play a minor role in most practical cases. The neglect of diffraction around the ends of barriers will introduce no significant errors, and it simplifies considerably the computational procedures. Furthermore, a diffracting barrier is then completely specified by the coordinates of the two endpoints of the top line defining the barrier segment.

The attenuation of sound by barriers is determined primarily by the difference, δ, between the path length of the shortest ray from the source over the top edge of the barrier the receiver and the path length of the direct ray from the source to the receiver in the absence of the barrier (Figure A-2).

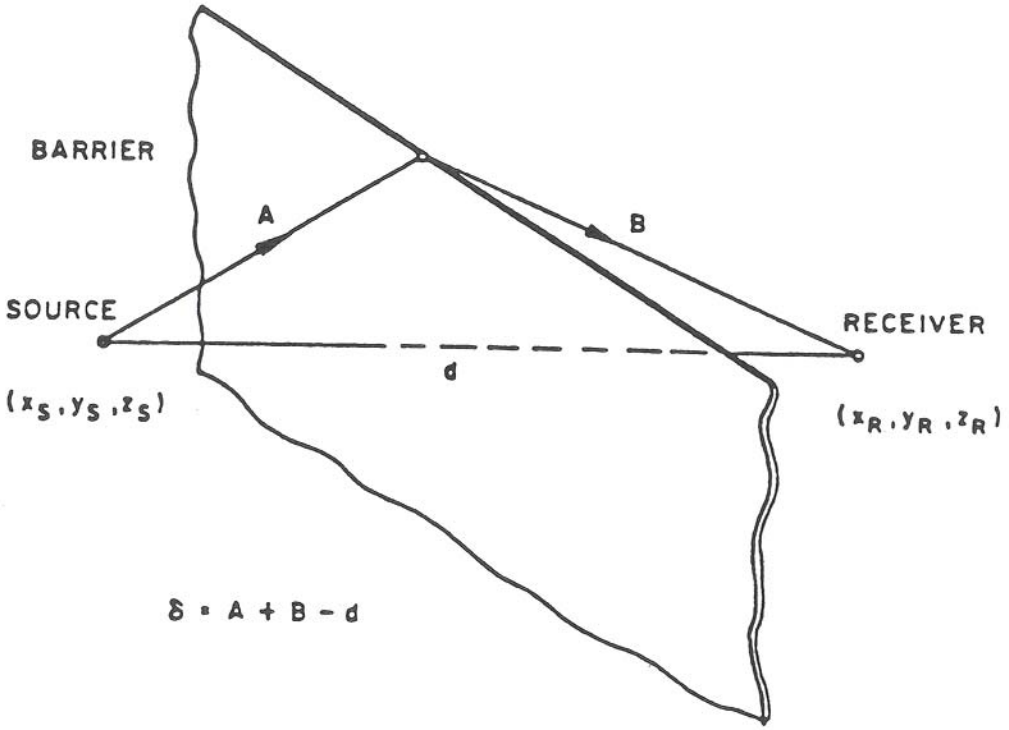


Figure A-2 DEFINITION OF THE PATH LENGTH DIFFERENCE δ FOR SOUND DIFFRACTION BY A BARRIER

For large path length differences, the attenuation in the acoustical shadow zone of a barrier is limited by effects of refraction and scattering of sound in the atmosphere. Based on data (5), the coded procedures have a maximum attenuation of 20 dB.

The attenuation for a barrier is not zero for zero path length difference (i.e., for a ray grazing over the barrier). For this situation, the theory of Fresnel diffraction yields an attenuation of about 5 dB. The attenuation becomes negligible when a direct sound ray traveling from the source to the receiver passes far over the top edge of the barrier. To simplify computation, diffraction effects are no longer considered when the height difference between the direct ray and the top of the barrier is greater than 20 feet.

For height differences of 20 feet or less, the Fresnel number is expressed as:

$$N = \frac{2\delta}{\lambda}, \qquad \text{(A-17)}$$

Where δ is the path length difference and λ is the wavelength corresponding to the center frequency of an octave band. For normal atmospheric conditions, the speed of sound in air is assumed to be 1120 ft/sec/ Thus, for a center frequency f the Fresnel number becomes:

$$N = \frac{2f}{c}\delta = f\delta/560 \qquad \text{(A-18)}$$

The barrier attenuation is calculated as a function of the Fresnel number, using an analytic approximation to the measured data of Maekawa (6):

$$\varepsilon_B = 20 \cdot \log_{10}\left(\frac{\sqrt{2\pi N}}{\tanh \sqrt{2\pi N}}\right) + 5, \text{ dB for } N \geq -0.2 \qquad \text{(A-19)}$$

$$\varepsilon_B = 0 \qquad \text{otherwise}$$

Equation (A-19) is applicable to both positive and negative values of N. However, for the actual computation, the values of attenuation are calculated as a function of N using the following relationships. (See Subroutine BARFAC)

$$
\begin{aligned}
\varepsilon_B &= 0 && \text{dB for } N \leq -0.2 \\
\varepsilon_B &= 20 \cdot \log_{10}\left(\frac{\sqrt{2\pi|N|}}{\tan\sqrt{2\pi|N|}}\right) + 5 && \text{dB for } -0.2 < N \leq 0 \\
\varepsilon_B &= 20 \cdot \log_{10}\left(\frac{\sqrt{2\pi N}}{\tanh\sqrt{2\pi N}}\right) + 5 && \text{dB for } 0 < N \leq 5.03 \\
\varepsilon_B &= 20 && \text{dB for } N > 5.03
\end{aligned}
\qquad \text{(A-20)}
$$

The last line in Equation (A-20) accounts for the above mentioned upper limit to barrier attenuation.

As shown in Figure A-3, the attenuation of the A-weighted sound pressure level of typical passenger car noise is almost identical with the sound attenuation in the 500Hz band. Hence, the primary number important for the reduced attenuation of rays oblique to the top edge of the barrier (7).

For noise from a road segment and for a barrier at oblique angle to the road, the coded procedures find the path length difference $^{\delta}N$ for sound from the nearest point on the road segment affected by the barrier. Then, by assuming a monotonic variation of the path length difference from other points on the road, the extreme ends of the road segment are considered. If the path length differences, $^{\delta}1$ and $^{\delta}2$, for these endpoints differ from $^{\delta}N$ by more than a number that results in an attenuation difference of about 1dB, the road segment between the near point N and the point 1 or point 2, respectively, is cut in half. New path length differences are calculated for the new end points of the road segment, and the procedure of reducing the length of the road segment is repeated until the attenuation by diffraction is approximately constant.

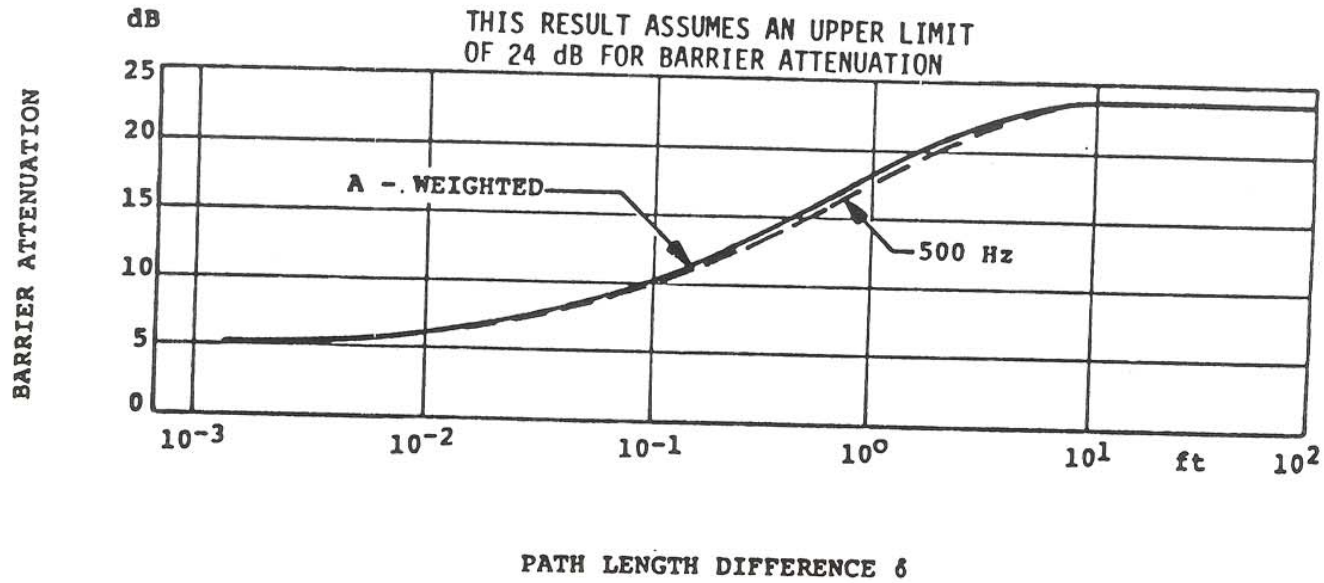


Figure A-3. BARRIER ATTENUATION AS A FUNCTION OF THE PATH LENGTH DIFFERENCE δ FOR A FREQUENCY OF 500 Hz AND FOR A-WEIGHTED LEVEL OF TYPICAL PASSENGER CAR NOISE

The criterion used for the acceptance of a sufficiently small difference in path length differences (i.e., uniform attenuation) is:

$$|\delta_2 - \delta_1| - \frac{\delta_1 + \delta_2}{100}\left(1 + \frac{\delta_1 + \delta_2}{2}\right) \leq 0.1 \qquad \text{(A-21)}$$

The numbers are based on a frequency of 500 Hz, for which the effect of Equation (A-21) on the attenuation is plotted in Figure A-4.

In case of multiple diffraction by several barriers in parallel, the coded procedures consider the strongest diffraction exclusively.

This is a conservative procedure resulting in attenuations that are somewhat too small, but it seems to be the most reasonable way to bypass the very complicated and not yet fully understood problem of multiple diffraction.

A.7 Reflection

The sound field at a receiver results from contributions of direct (or diffracted) and reflected rays. In many practical cases of sound propagation from a highway, corrections applied for reflections are small compared to the inaccuracies involved in the prediction of ground attenuation and in uncertainties with acoustical shadow zones owing to wind and temperature gradients in the atmosphere. Therefore, the model has been designed to account for reflections with a first-order approximation.

The reflection model utilized by the traffic noise prediction code disregards phase relations between the various contributions and considers incoherent waves for which the total sound intensity is the sum of the intensities of the individual contributions.

Reflections from the road surface are always present. However, the contributions from these reflections are implicitly included in references levels at a short distance from individual vehicles on the road.

Reflections at the ground plane farther from the roadway are disregarded because they generally result in a complex interference pattern with the direct ray. Consideration of these effects is beyond the scope of a first-order approximation for reflections.

Reflections at any inclined plane result in rays directed either toward the ground, and thus being neglected, or toward the sky, and thus not contributing to the sound intensity at a normal receiver location close to the ground. Therefore, only reflections on plans that are perpendicular to the ground plane are considered by the prediction code.

Within the first-order approximation, this model also neglects the actual frequency-dependent magnitude of reflection coefficients and distinguish only between reflection coefficients 0 and 1 of reflecting surfaces (i.e., perfect absorption or perfect reflection, respectively).

In order to determine whether a reflective barrier is high enough to be effective, the procedures consider the possibly reflected ray that travels a minimum distance from the road segment to the receiver. A reflective plane perpendicular to the ground is considered high enough if the direct ray strikes the barrier at least 2 feet below the top edge of the barrier. For reflection points within 2 feet of the top edge, diffraction effects are considered by the model to be strong enough for all frequencies so that the reflected ray is sufficiently reduced in amplitude to be negligible.

Also neglected by the model are reflections from planes that are either very short or very remote so that the contribution to the sound intensity at the receiver is less than 10 percent of the intensity received via direct (or diffracted) rays from the road segment under consideration. The analytical formulation for this criterion is:

$$\frac{d\Delta\alpha'}{d'\Delta\alpha}\ 10^{-\varepsilon_B/10} < 0.1 \qquad \text{(A-27)}$$

where d = distance from the receiver to the road segment in feet

$\Delta\alpha$ = aspect angle of the road segment at the receiver

d' = distance from the road segment to a receiver location imaged about the reflector

$\Delta\alpha'$ = aspect angle of the barrier at the image receiver

ε_B = Attenuation in dB by diffraction of the direct ray due to a possible barrier (referred to a frequency of 500 Hz)

Single reflections are considered exclusively; contributions from rays that strike two or more reflectors are ignored. It is essential for the future calculation of higher order statistical parameters of road traffic noise that the reflections of sound from a certain road segment be treated as amplifications of the direct (or diffracted) rays and not as uncorrelated contributions from independent road segments. The factor F multiplying the intensity of the direct sound from a road segment is calculated in Subroutine GEOMRY using

$$F = 10^{-\varepsilon_B/10} + \sum_{i=1}^{N} \frac{\Delta\alpha_i' d}{\Delta\alpha d_i'}\ 10^{-\varepsilon_i/10} \qquad \text{(A-28)}$$

where the subscript i indicates reflections at N different surfaces, each of which might be diffracted by a barrier before or after reflection and therefore might have and attenuation ε_i . The factor F is calculated as a function of frequency. The notation in Equation (A-28) is the same as in Equation (A-27) except for the angle $\Delta\alpha'$ which denotes the aspect angle of the road segment at the image receiver (Figure A-7).

A.8 Combination of Attenuation and Reflection

A.8.1 Atmospheric Absorption

The overall procedures account for atmospheric absorption in combination with barrier diffraction, ground absorption and reflections. The path length used for calculating the atmospheric absorption is the direct distance from the source to the receiver and is not corrected for the path length difference δ of diffracted rays nor for the increased path length of reflected rays. The factor F, defined by Equation (A-28) is multiplied by $10^{-D_A/10}$, where D_A is defined by Equation (A-16), in order to calculate, for each individual road segment, the factor

$$10^{-\varepsilon/10} = F \cdot 10^{-\varepsilon_A/10} \cdot 10^{-\varepsilon_G/10} \qquad \text{(A-29)}$$

This composite attenuation factor is employed for the calculation of the energy mean level in Equations (A-8) and (A-9).

A.8.2 Ground Absorption

The prediction procedure includes ground aborption if such attenuation is significant only for the case of no diffraction of sound from the source to the receiver. That is, if in analyzing a sub-segment of a roadway segment, diffraction of the direct ray from the source to the receiver is encountered, the prediction code ignores the attenuation resulting from ground cover of all types for the sub-segment of roadway being analyzed. As indicated by Equation (A-29), the same ground absorption is assumed for both the direct rays and for all reflections.

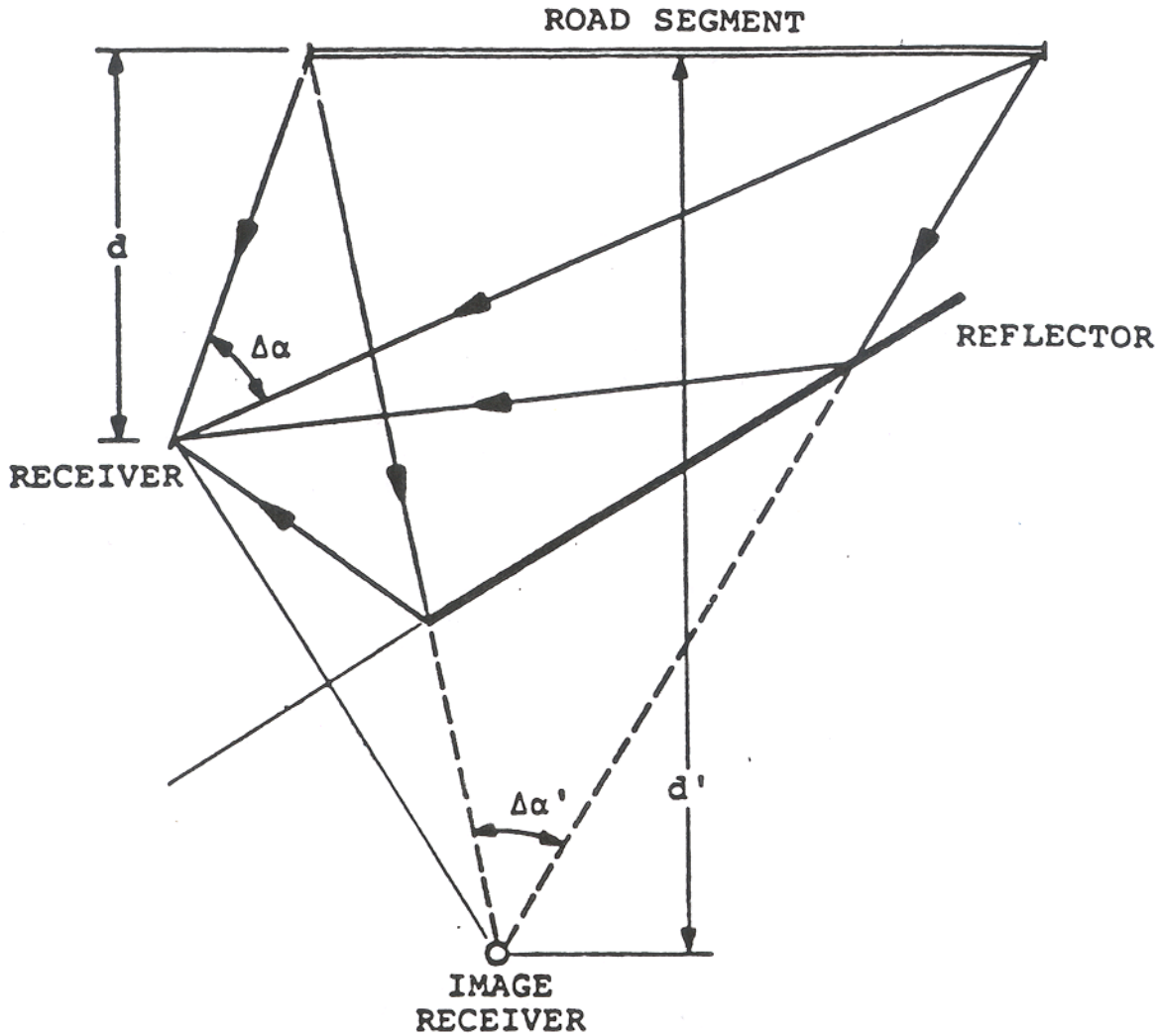


Figure A-7 EXAMPLE FOR RAY TRACES FROM A REFLECTOR, INDICATING THE CONSTRUCTION BY MEANS OF AN IMAGE RECEIVER

A.8.3 Reflection Before or After Diffraction

The procedures account for reflections in combination, with diffraction provided that there is only a single diffraction before or after the reflection and that the path length increase due to diffraction is less than 5.6 feet. Doubly diffracted reflections are neglected as well as very weak single reflections that suffer, in the 500 Hz band, the maximum attenuation of 20 dB assumed for barrier diffraction.

The attenuation of reflected rays by diffraction is calculated for one location on the road segment only: the point nearest to the image receiver. No attempt is made to refine this calculation by checking for the attenuation from other points on the road segment, since the contribution of diffracted reflections will be generally small and, hence, inaccuracies of the calculation will be negligible.

A.9 OPTIMA Effectiveness/Cost Ratios

OPTIMA calculates so-called effectiveness/cost ratios for each barrier section at each height to help the user design cost-efficient barrier systems. This section presents first a mathematical description of how the E/C ratios are derived, and second, a discussion of why the ratios are defined as they are (4).

Mathematical Description

Actually, what the user sees in the matrix (table) of so-called E/C ratios are the ratios of the change in effectiveness to the change in cost for each height of each barrier section. Therefore, the E/C ratios could be more accurately called E/C ratios. The ratios are the increase in effectiveness divided by the increase in cost brought abut by the increase in height

For each barrier section. To compute these changes at a specified height, the values of effectiveness and cost are computed at the specified height, z, and at heights greater than (z+DELZ) and less than (z-DELZ) the specified height. Changes in these values are then determined and averaged.

The effectiveness, E_b,i, of barrier section b, at height i, is based on the population-weighted, DNL-weighted sound energy that goes over the top of the barrier summed over all receivers, W_b,i.

Set $D_r = 67$

$$W_{b,i} = \sum_{r=1}^{R} P_r 10^{\left(\frac{L_{b,i,r}}{10}\right)} 10^{\left(\frac{67-D_r}{10}\right)} , \qquad \text{(A-30)}$$

where P_r = number of people represented by receiver, R

$10^{\left(\frac{L_{b,i,r}}{10}\right)}$ = sound energy passing over barrier b, at height i, to receiver r

D_r = DNL at receiver r

R = total number of receivers

b = index over barrier sections

i = index over barrier section heights

r = index over receivers

The effectiveness at height i is defined as the difference between the weighted sound energy at height i and the weighted sound energy at the ground height (i=1). So,

$$E_b,i = -(W_b,i - W_b,1) \qquad \text{(A-31)}$$

We want to determine the <u>change</u> in effectiveness with respect to the change in cost at barrier height i, so the first step is to determine the increase in effectiveness from height i to height i+1. That is,

$$\Delta E_{b,i} = E_{b,i+1} - E_{b,i} \qquad \text{(A-32)}$$

Now,

$$E_{b,i+1} = -(W_{b,i+1} - W_{b,1}) \qquad \text{(A-33)}$$

And substituting (A-31) and (A-33) into (A-32), we get

$$\Delta E_{b,i} = -(W_{b,i+1} - W_{b,1}) - [-(W_{b,i} - W_{b,i})]$$

$$\Delta E_{b,i} = -(W_{b,i+1} - W_{b,i}) \qquad \text{(A-34)}$$

So, the change in effectiveness from height i to height i+1 is based only on the weighted sound energy at height i and height i+1, and not on the energy at the ground height.

Now, the total barrier section cost, C_b,i, is just the cost per linear foot A_b,i for height i, multiplied by the length of the barrier, S_b, or

$$C_{b,i} = A_{b,i} S_b \quad .$$

And, the change in cost, C_b,i, is

$$\Delta C_{b,i} = C_{b,i+1} - C_{b,i} = S_b(A_{b,i+1} - A_{b,i}) \qquad \text{(A-36)}$$

So, the ratio of the change in effectiveness to change in cost for a change in height from i to i+1 is

$$\Theta_{b,i} = \frac{\Delta E_{b,i}}{\Delta C_{b,i}} \qquad \text{(A-37)}$$

If we substitute Eq. (A-30) for heights i and i+1 in Eq (A-34), and the resulting (A-34) and (A-36) into (A-37), we get

$$\Theta_{b,i} = \frac{-\left[\sum_{r=1}^{R} P_r 10^{\left(\frac{L_{b,i+1,r}}{10}\right)} 10^{\left(\frac{67-D_r}{10}\right)} - \sum_{r=1}^{R} P_r 10^{\left(\frac{L_{b,i,r}}{10}\right)} 10^{\left(\frac{67-D_r}{10}\right)}\right]}{S_b(A_{b,i+1}-A_{b,i})} \qquad \text{(A-38)}$$

But Eq. (A-38) gives θ only for a change in barrier section height from i to i+1. What is needed is the value of θ <u>at</u> height i. OPTIMA determines the ratio of change in effectiveness to change in cost at height i by averaging $\theta_{b,i}$ and $\theta_{b,i-1}$ (the ratio of change in effectiveness to change in cost for a change of height from i-1 to i). Specifically the expression used to determine the ratio at height i, $\xi_{b,i}$, is

$$\xi_{b,i} = 10 \log \left[\Theta_{b,i}\Theta_{b,i-1}\right]^{1/2} \qquad \text{(A-39)}$$

At the maximum barrier height, where i - n, $\theta_{b,n}$ cannot be computed since no n+1 height exists. Therefore, $\theta_{b,n-1}$, which is based on W_n and W_n-1, is used to compute $\xi_{b,n}$ as follows:

$$\xi_{b,n} = 10 \log \Theta_{b,n-1} \qquad \text{(A-40)}$$

Thus, Eqs. (A-38), (A-39), and (A-40) completely define the matrix that contains the ratios of changes in effectiveness to changes in cost for each height of each barrier section. This is an array of manageable dimensions (bxi) that can be easily scanned to determine relative cost effectiveness for each barrier section. Choosing a system of barrier section-heights that have equal or nearly equal values of $\xi_{b,i}$ means the resultant barrier system will be balanced with respect to cost and effectiveness.

Why $\Delta E/\Delta C$?

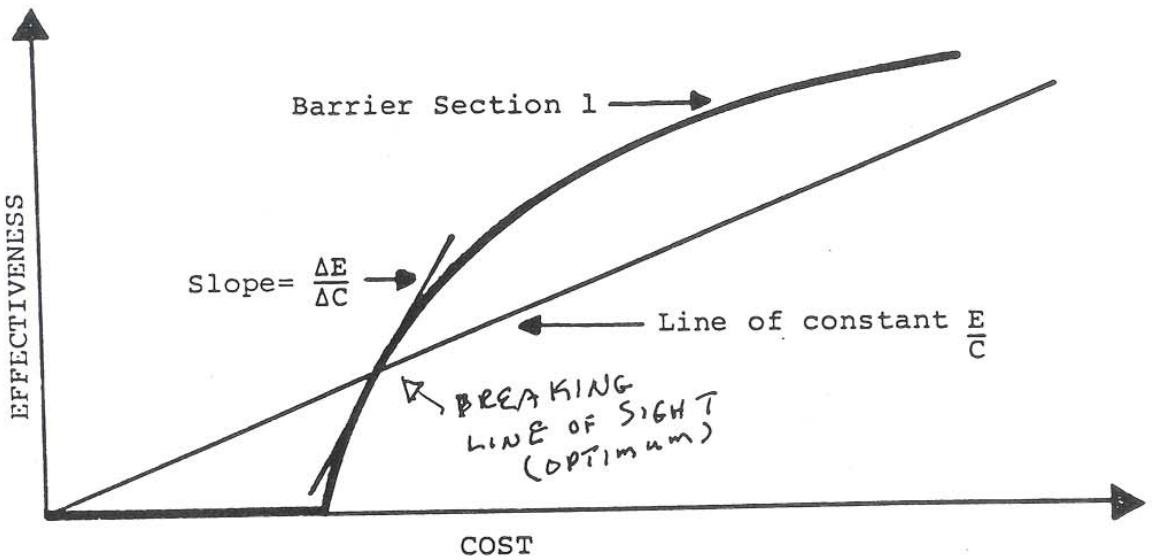
OPTIMA calculates and tabulates ratios of the change in effectiveness to the change in cost, and not simply the ratio of the effectiveness to cost. This section describes why by way of example.

Figure 1 shows two graphs of effectiveness vs. cost for two different sections of the same barrier. Section 1 has to be built up several feet before it breaks the line-of-sight to the receivers, therefore, some cost is attributed to the barrier before any effectiveness is realized. Section 2 is at the top of a small cut and therefore, the barrier exhibits effectiveness as any cost above zero is applied.

In addition to the heavy curve indicating the path the barrier follows as height is increased, is a straight line shown on both graphs. The line represents a constant ratio of effectiveness to cost. At the point where the lines intersect the barrier curves. the slope of the curve is shown with a straight line segment. The slope of the curve indicates the change in effectiveness with respect to the change in cost, or $\Delta E/\Delta C$ for that ratio of effectiveness to cost (E/C).

Note that the slope of the curve for barrier section 1 is much steeper than that for section 2. This means that for a given change in cost, there will be a much more significant change in effectiveness for barrier section 1 than for section2. The conclusion that one can draw here is that significant cost savings could result with no overall chance in effectiveness if money were transferred from barrier section 2 to section 1. That is because for section 2, a a large cost reduction can be realized with only a small loss in effectiveness by moving back on the curve, toward the origin. That loss in effectiveness can be made up by barrier section 1 with only a small increase in cost by moving slightly up the section 1 curve. Therefore, the only situation in which a money transfer would not be advantageous, is one in which the slopes are equal. When the slopes are equal, $\Delta E_1/\Delta C_1 - \Delta E_2/\Delta C_2$, and the barrier system is said to be "balanced" with respect to cost and effectiveness.

Only the situation where the slopes of effectiveness/cost ratios for each barrier section are equal ensures a balanced barrier system.



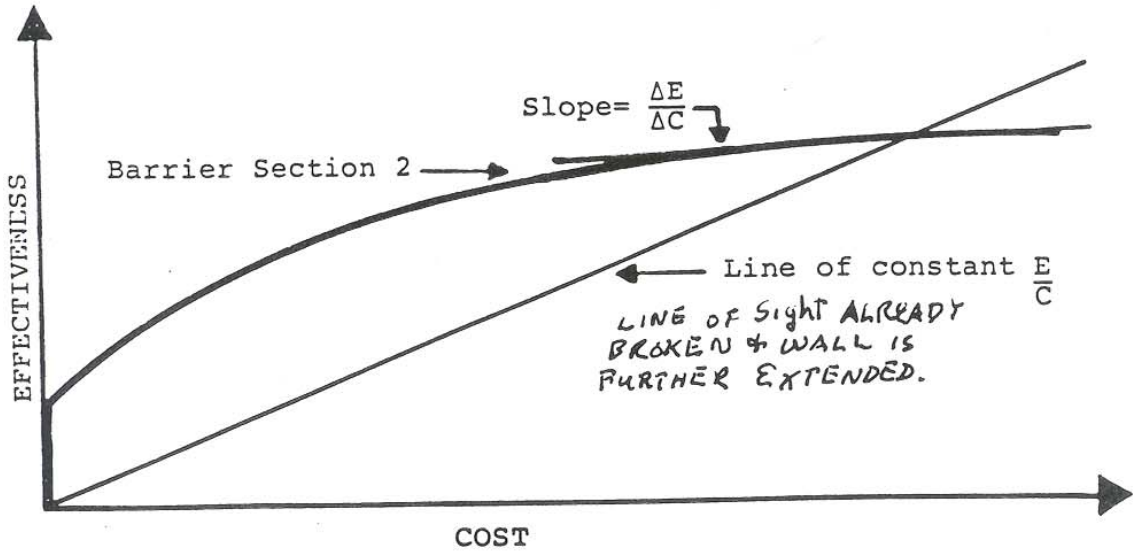


FIGURE 1. EFFECTIVENESS VS. COST FOR TWO SECTIONS OF THE SAME BARRIER