Development of a Quick Reliability Method for Mechanistic-Empirical Asphalt Pavement Design

Submission date: August 1, 2001
Word Count: 4654

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ABSTRACT

In Mechanistic-empirical (M-E) pavement design, the Monte Carlo method has proven to be an effective means of determining reliability. One drawback is the amount of computing time required to carry out a simulation. Performing a Monte Carlo simulation on a complex load spectrum can require hours on a typical desktop computer. The Minnesota Department of Transportation is developing a shortcut method for estimating reliability in its MnPAVE flexible pavement design program. In this program, variability in the layer thicknesses and moduli are expressed as coefficients of variation (CV). Development of the shortcut method involves running a large number of pavement design simulations to generate damage factors (using Miner's Hypothesis) and reliability values (using the Monte Carlo method). For a given set of CV values, there is a strong correlation between the damage factor and reliability. For the two asphalt fatigue models tested, this correlation exists regardless of the number and thickness of pavement layers, layer moduli, number of seasons, and number of axle configurations. For the rutting model the addition of more seasons and loading configurations increases scatter in the data. A family of curves can be generated to include the range of expected CV combinations, and the resulting equations can be used to estimate the reliability of a pavement design based on a single damage calculation.

INTRODUCTION

While reliability has been a concept used in pavement design for many years, the recent emphasis on mechanistic-empirical (M-E) design has added new urgency to finding an efficient way of calculating the reliability of a pavement design. Determining reliability requires quantifying the variability of the input values (such as layer thickness and modulus) and then using those values to estimate the variability of the output (expected pavement life). M-E design procedures typically use a numerical method (layered-elastic analysis or finite element) to simulate the pavement structure and its response to traffic loads. The complexity of these methods precludes the use of analytical calculations to arrive at reliability, so a numerical method must be selected.

The Monte Carlo method has proved to be a versatile means of estimating the variability of an output distribution based on the respective coefficients of variation (CV) of the input distributions. This is done through repeated simulations using input values randomly selected from their respective probability distributions. After a sufficient number of repetitions have been completed, the reliability can be estimated from the output distribution. The main drawback to the Monte Carlo method is the time required to complete the necessary number of simulations. The required time can range from minutes to hours depending on the complexity of the problem. As a result, it may slow the implementation of an M-E design program.

MnPAVE is an M-E asphalt design procedure under development at the Minnesota Department of Transportation (Mn/DOT). Included in this procedure is a Windows-based software program based on layer-elastic analysis (LEA) of a pavement structure. The LEA portion of the software is based on the WESLEA model developed by Van Cauwelaert et al. (*1*). In the MnPAVE software, the user enters climate, structural, and traffic data. The LEA model calculates stresses and strains at critical points in the pavement structure. Transfer functions and Miner's Hypothesis are then used to determine the damage factor and expected life of the pavement.

MnPAVE SOFTWARE

Input Values

Structural Input

In order to calculate damage and reliability, the following inputs are required for each layer in the pavement structure:

1. Thickness (H)
2. Thickness coefficient of variation (HCV)
3. Elastic modulus (E) during each season
4. Modulus coefficient of variation (ECV) and distribution type.
5. Poisson's Ratio

The coefficient of variation is simply the standard deviation divided by the mean value and is expressed as a percentage. Timm et al. (*2*) determined that layer thickness variability can be described by a normal distribution, and layer modulus variability by a lognormal distribution.

Traffic Input

Currently, there is a choice between using Equivalent Single Axle Loads (ESALs) or load spectra. An ESAL is defined as a 80 kN (18 kip) dual tire axle load. The load spectrum consists of a combination of single and dual tires in single, tandem, or tridem axle configurations. Axle loads are organized in 8.9 kN (2.0 kip) increments. Additionally, the tire pressure is required in order to calculate the load radius.

Layered-Elastic Analysis Output

The LEA model calculates normal stresses, strains, and deflections as well as shear stresses at any point in the pavement structure. In MnPAVE, critical strains are used to determine damage and reliability. The critical strains are the tensile strain at the bottom of the asphalt layer and the compressive strain at the top of the subgrade.

Transfer Functions

Fatigue

Two fatigue models are currently being tested for MnPAVE. Timm et al. (*2*) used a version of the "Illinois" equation developed by Thompson (*3*) that was calibrated using data from the Minnesota Road Research Project (Mn/ROAD). The Mn/ROAD version is shown in Equation 1.

$$N_f = 2.83 \times 10^{-6} \left(\frac{1}{\varepsilon_t} \right)^{3.20596} \qquad (1)$$

where

N_f = Number of load repetitions required to reach fatigue failure.

ε_t = Horizontal strain at the bottom of the asphalt layer.

A fatigue model developed by Finn et al. (*4*) was also evaluated. This model (Equation 2) includes a modulus term for the asphalt layer in order to capture the relationship between stiffness and fatigue cracking.

$$N_f = 18.4 \left(0.00432 \varepsilon_t^{-3.291} E^{-0.854} \right) \qquad (2)$$

where

E = Modulus of the asphalt layer (psi).

Rutting

Equation 3 shows the rutting transfer function used in MnPAVE. This equation was also calibrated for Mn/ROAD as described by Timm et al (*2*).

$$N_R = 5.5 \times 10^{15} \left(\frac{1}{\varepsilon_v \times 10^6} \right)^{3.949} \qquad (3)$$

where

N_R = Number of load repetitions required to reach rutting failure.

ε_v = Vertical strain at the top of the subgrade

Miner's Hypothesis

Central to the MnPAVE software is the calculation of lifetime pavement damage using Miner's Hypothesis (*5*). In the simplest case (single load configuration and no seasonal variations in material properties), the damage over the life of the pavement can be characterized by Equation 4:

$$Damage = \frac{n}{N} \qquad (4)$$

where

Damage = an index indicating the expected level of damage after *n* load applications (*Damage* ≥ 1 indicates pavement failure)

n = applied number of loads

N = number of loads required to cause failure (based on empirical transfer functions)

In order to model the actual conditions expected over the life of a pavement, multiple seasons and load configurations can be introduced. In this case, Equation 1 is not sufficient, as *N* is based on the response to a single load configuration on a pavement with single set of seasonal material properties. In this case, Miner's Hypothesis can be used to calculate damage (Equation 5).

$$Damage = \sum_{i=1}^{k} \sum_{j=1}^{m} \frac{n_{season_i, load_j}}{N_{season_i, load_j}} \qquad (5)$$

where

$n_{season_i, load_j}$ = number of applications of load *j* that occur during season *i*.

$N_{season_i, load_j}$ = number of applications of load *j* during season *i* required to cause failure.
k = total number of seasons.
m = total number of axle load configurations.

Reliability

MnPAVE uses a modified version the Monte Carlo method described by Timm et al. (*2*). A flow chart representation of the MnPAVE procedure is shown in Figure 1. The steps in the MnPAVE Monte Carlo simulation are as follows:

1. Randomly select input values from their respective probability distributions.
2. Calculate the damage using Miner's Hypothesis.
3. Perform enough cycles to generate a repeatable output distribution (2,000 cycles provides sufficient repeatability in MnPAVE).
4. Determine the number of cycles that resulted in *Damage* < 1.
5. Calculate reliability according to Equation 6:

$$Reliability = 100 \times \frac{\text{number of cycles where } Damage < 1}{\text{total number of cycles}} \quad (6)$$

The method of selecting the input values for each cycle differs according to the shape of the input distribution. The procedure is described in detail by Timm et al. (*2*). The first step requires the generation of a pair of independent standard normal variates as shown in Equations 7 and 8.

$$S_1 = \sqrt{-2 \times \log(U_1)} \times \sin(2\pi \times U_2) \quad (7)$$

$$S_1 = \sqrt{-2 \times \log(U_1)} \times \cos(2\pi \times U_2) \quad (8)$$

where

S_1, S_2 = standard normal variates.
U_1, U_2 = independent standard uniform variates (random numbers between 0 and 1)

Next, thickness values can be generated for the first two layers from their respective normal distributions as shown in Equations 9 and 10.

$$H_1 = \mu_1 + \sigma_1 S_1 \quad (9)$$

$$H_2 = \mu_2 + \sigma_2 S_2 \quad (10)$$

where

H_1, H_2 = randomly generated thickness values for layers 1 and 2
μ_1, μ_2 = mean values of layers 1 and 2
σ_1, σ_2 = standard deviation of layers 1 and 2 ($\sigma = \mu \times$ CV/100)

Equations 7 through 10 can then be repeated for layers 3 and 4. For lognormally distributed modulus values, Equations 7 and 8 can again be used to generate S_1 and S_2. For a lognormal variable *E* and transformed variable *Y* = ln(*E*), Equations 11 and 12 can be used to calculate the standard deviation and mean of the transformed variable, respectively.

$$\sigma_y = \sqrt{\ln\left[\left(\frac{\sigma_E}{\mu_E}\right)^2 + 1\right]} \quad (11)$$

$$\mu_y = \ln(\mu_E) - \frac{\sigma_y^2}{2} \quad (12)$$

Equations 13 and 14 can then be used to generate pairs of modulus values.

$$E_1 = e^{\mu_y + \sigma_y S_1} \quad (13)$$

$$E_2 = e^{\mu_y + \sigma_y S_2} \tag{14}$$

MnPAVE SIMULATIONS

A preliminary analysis of fatigue results based on the Mn/ROAD fatigue transfer function (Equation 1) indicated that the relationship between damage and reliability as calculated in Equation 6 is insensitive to the number of seasons or axle load configurations. For this reason, the vast majority of simulations were conducted using a single season and load configuration in order to minimize the computing time. A variety of pavement structures were simulated in order to test the robustness of the damage-reliability relationship. Structures ranged from 4-layer pavements to full-depth asphalt pavements with varying thickness and moduli. Due to the limited scope of the current study, only three combinations of CV values were tested. They are designated as High, Medium, and Low, and are described in Table 1.

RESULTS

Regression Equations

The damage-reliability plots are shown in Figures 2 through 4. A regression equation of the form shown in Equation 15 was fit to each set of data. In some cases, this equation has a positive slope for damage values near 0 (where Monte Carlo reliability = 100%) and also exceeded 100%. For this portion of the data, a conditional statement was inserted to set the value equal to 100%.

$$R = e^{aD^3 + bD^2 + cD + d} \tag{15}$$

where

R = predicted reliability, %
D = calculated damage
a, b, c, d = regression coefficients

Residual Analysis

Before running a residual analysis, the 100% reliability values generated by the conditional statement were removed to avoid skewing the results. To analyze nonlinear regressions, Devore (*6*) plotting residuals (e) and standardized residuals (e*) vs. the predicted values. Figures 5, 6, and 7 show these plots for Mn/ROAD Fatigue (High CV), Finn Fatigue (Medium CV), and Mn/ROAD Rutting (Low CV). The standard deviation of the residuals (s) provides an indication of the scatter. These values are shown in Table 2. According to Devore (*6*), the residuals are expected to be normally distributed which means that approximately 95% of the residuals should lie between ± 2 standard deviations (standardized residuals should lie between ± 2.0). These percentages are also shown in Table 2. The values range from 92.9 to 96.0, which provides confidence that the residuals are normally distributed.

Mn/ROAD Fatigue

After viewing Figure 2, it is clear that the Mn/ROAD fatigue model resulted in the best fit for the damage-reliability relationship. The three CV levels are clearly differentiated, and the results from multiple season simulations and load spectra appear to be no different than those of single season, single load simulations. Mn/ROAD fatigue had lower s-values (Table 2) than the other two models for all levels of CV, and the High CV case had the lowest s-value (1.12). This indicates both that this equation is a good predictor for Mn/ROAD fatigue.

Finn Fatigue

The higher s-values in the Finn fatigue data (Table 2) indicate a greater degree of scatter and/or a poorer fit than the Mn/ROAD fatigue model. However, multiple season and load spectra results still agree well with the single season, single load data. Compared to Mn/ROAD fatigue, the Finn fatigue s-values increase more significantly in the low CV cases. The plots in Figure 3 indicate that much of this increase is due to a poorer fit of the regression equation.

Mn/ROAD Rutting

Increasing the complexity of the design (multiple seasons and loads) appears to increase the scatter and the data points from adjacent sets tend to overlap. Despite this it is important to note that the Mn/ROAD rutting s-values (Table 2) are reasonably low, and the regression curves for the Mn/ROAD rutting data (Figure 4) indicate a clear difference between the three levels of CV.

CONCLUSIONS

1. The Mn/ROAD fatigue model shows the most promise in terms of developing a quick predictive model for pavement reliability.
2. Many more simulations are required to encompass the full range of pavements types and CV values for both thickness and modulus.
3. While the Mn/ROAD rutting results show less promise in their current form, better relationships may be possible if different pavement types are considered separately.
4. The selected regression equation did not fit the Finn fatigue data as well as the other two models. Further refinement may be necessary.
5. A neural network may prove useful in developing a comprehensive predictive model for pavement reliability.
6. A quick method of calculating reliability will greatly speed up the process of evaluating preliminary pavement designs. It may still be desirable to run a full Monte Carlo simulation to verify the final design.

ACKNOWLEDGMENTS

The author would like to acknowledge Shongtao Dai, John Siekmeier, and Dave Van Deusen at the Minnesota Department of Transportation for the technical expertise they provided. Ryan McKane and Peter Davich also deserve recognition for the hours they spent running simulations.

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TABLE 1 Coefficient of Variation Values (%) Used in MnPAVE Simulations

Property	High CV	Medium CV	Low CV
Hot-Mix Asphalt Modulus	30	20	10
Aggregate Base Modulus	40	30	20
Aggregate Subbase Modulus	40	30	20
Subgrade Soil Modulus	50	40	30
Layer 1 Thickness	5	5	5
Layer 2 Thickness	8	8	8
Layer 3 Thickness	15	15	15

TABLE 2 Regression Coefficients for Damage-Reliability Relationships

Model	Coefficients a	b	c	d	Std. Dev. of Residuals, s	Percentage of |e*| < 2s
Mn/ROAD Fatigue						
High CV	0.7417	-1.8536	0.2607	4.5988	1.12	93.3
Medium CV	-0.1197	-0.9568	0.2772	4.5944	1.59	94.4
Low CV	-1.7888	1.1991	-0.1905	4.6103	1.69	95.8
Finn Fatigue						
High CV	0.3355	-1.6286	0.4561	4.5824	1.79	94.3
Medium CV	-0.7487	-0.2538	0.2008	4.5912	2.40	94.6
Low CV	-2.3799	2.0228	-0.4297	4.6235	3.34	92.9
Mn/ROAD Rutting						
High CV	0.876	-1.4559	-0.2949	4.6267	2.23	95.3
Medium CV	0.9081	-1.8832	0.1394	4.6064	2.14	96.0
Low CV	0.4656	-1.6386	0.3776	4.5891	2.53	96.0

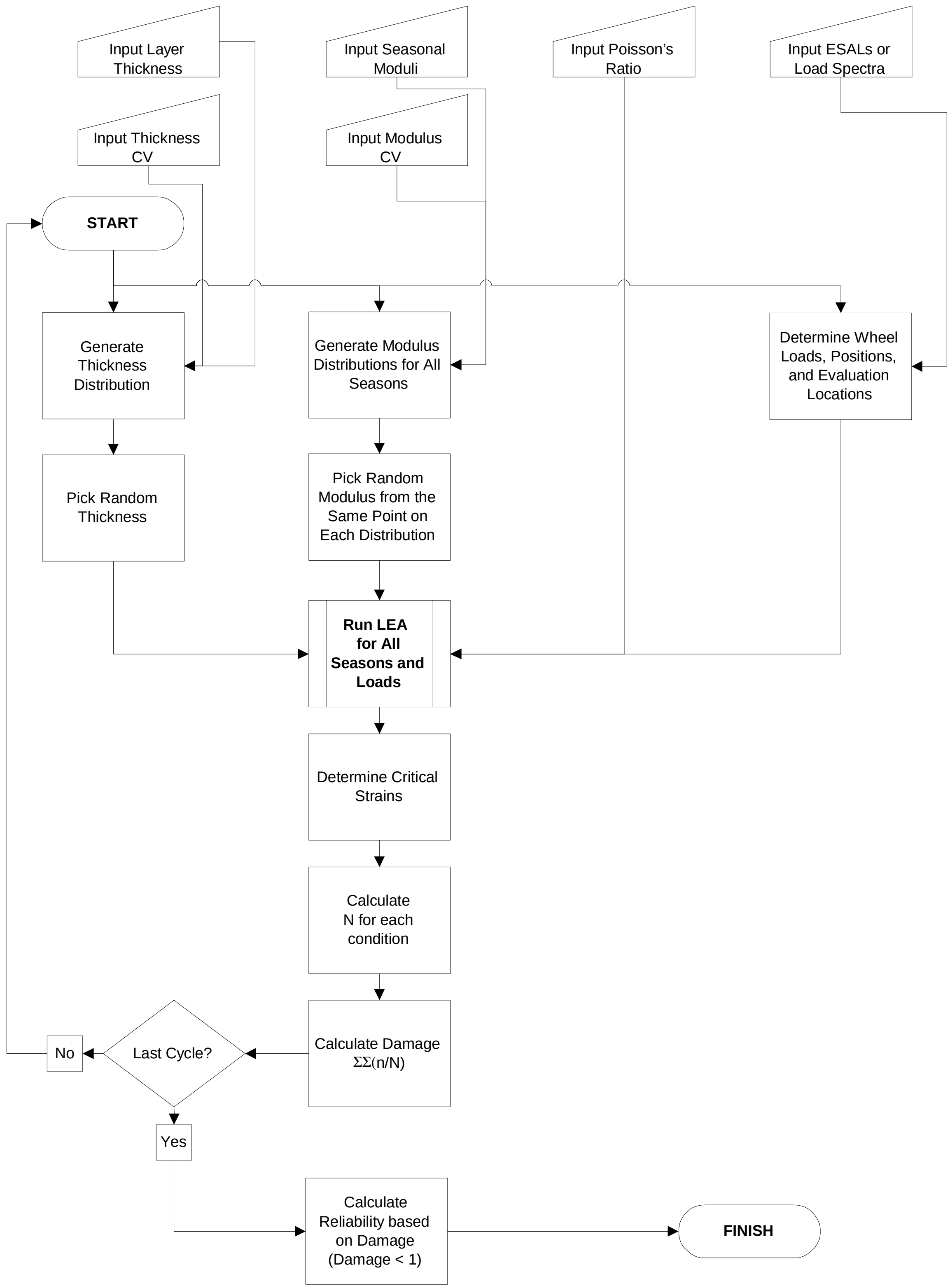

FIGURE 1 MnPAVE Flow Chart

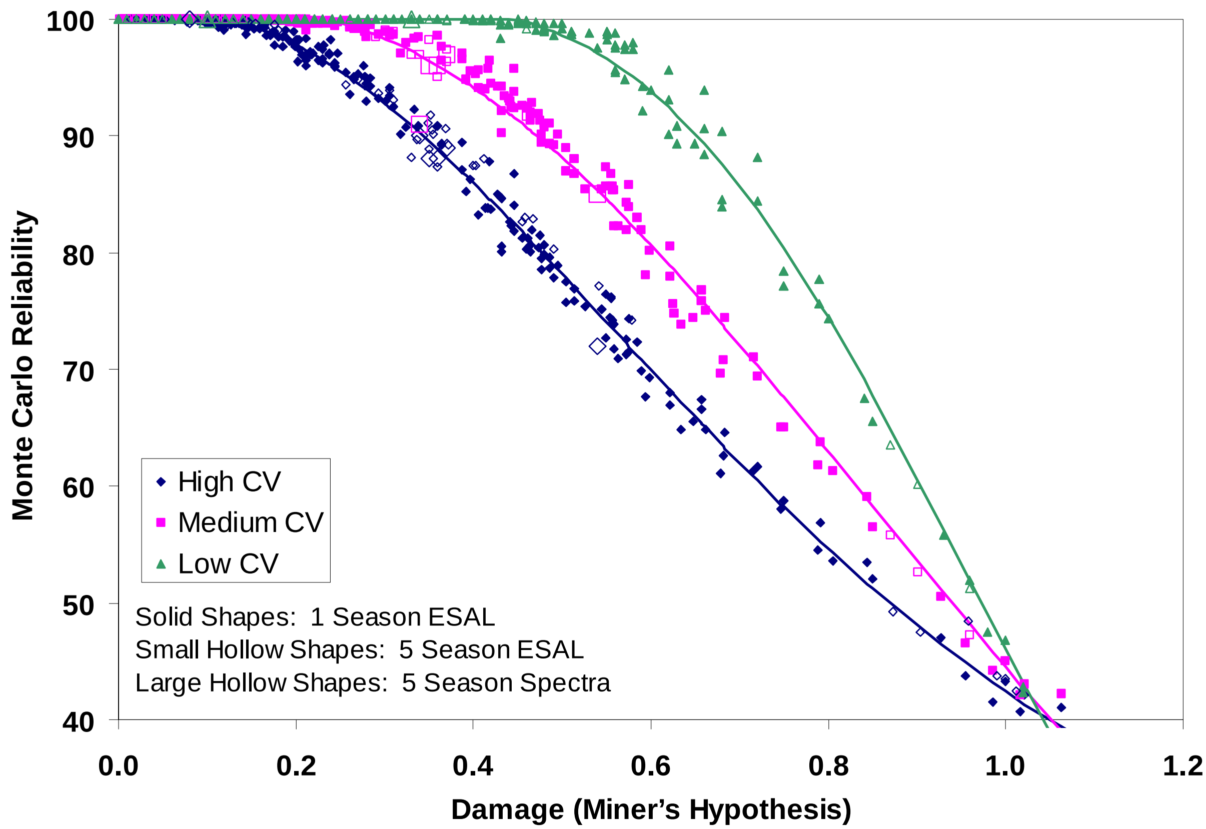

FIGURE 2 Reliabilty vs. Damage for the Mn/ROAD Fatigue Model

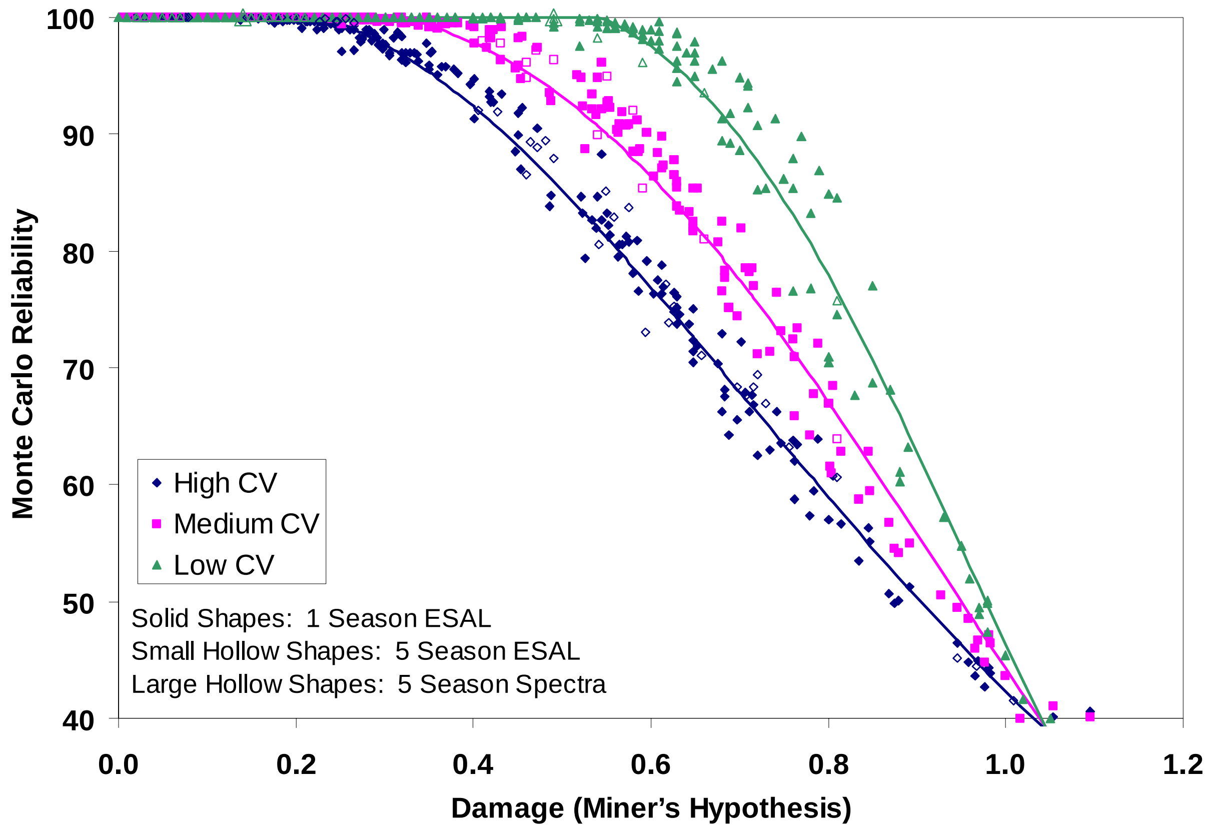

FIGURE 3 Reliability vs. Damage for the Finn Fatigue Model

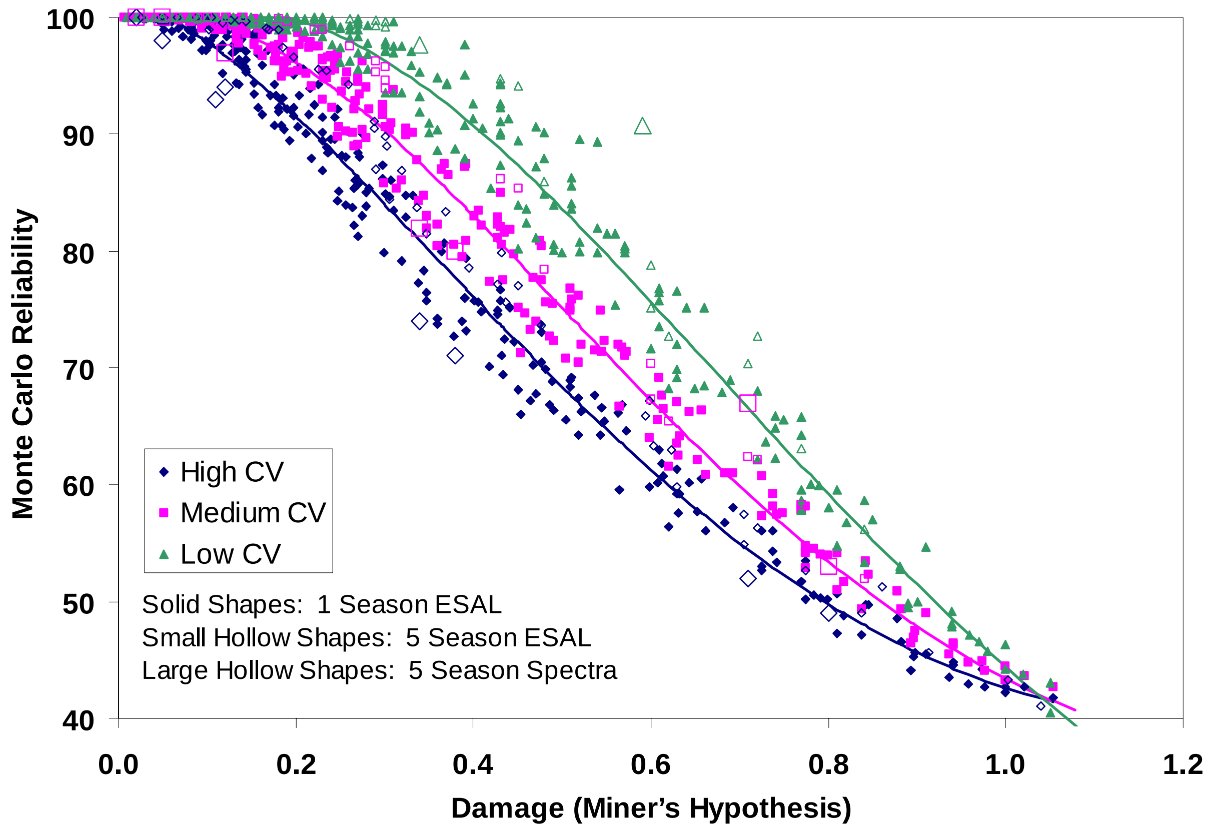

FIGURE 4 Reliability vs. Damage for the Mn/ROAD Rutting Model

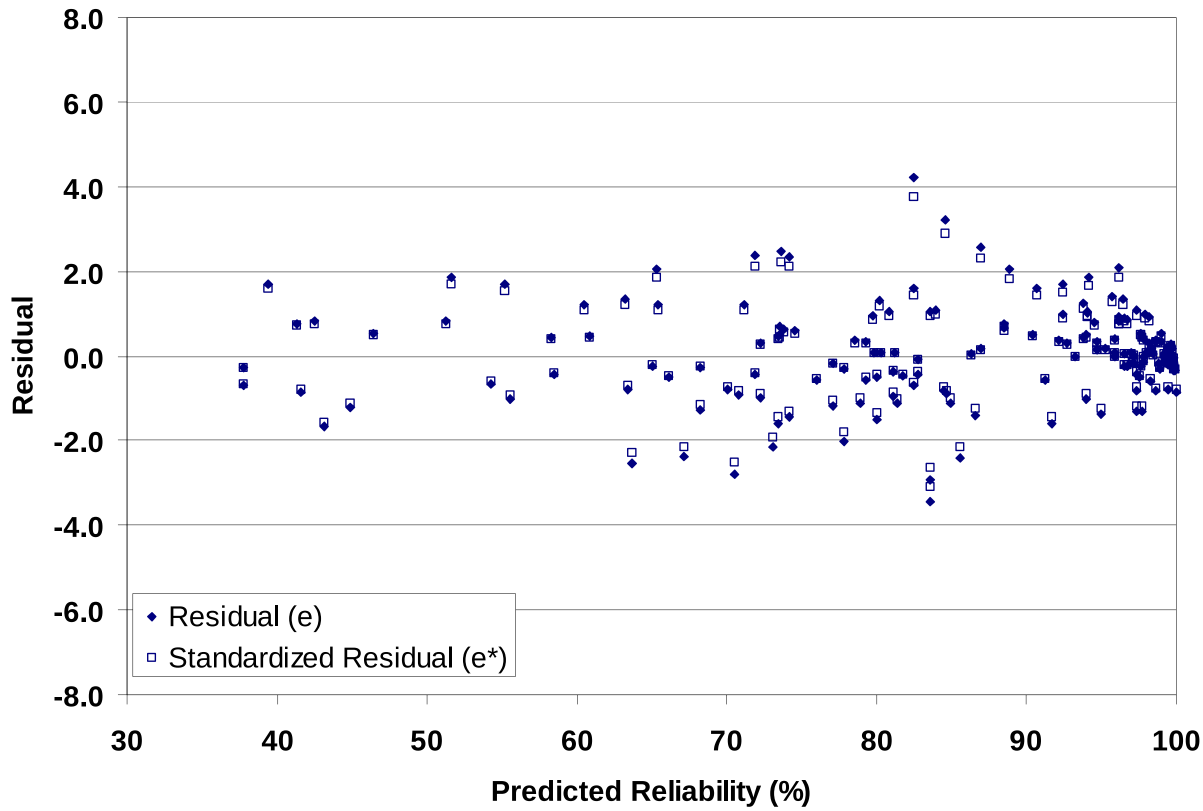

FIGURE 5 Residual Plot for Mn/ROAD Fatigue (High CV)

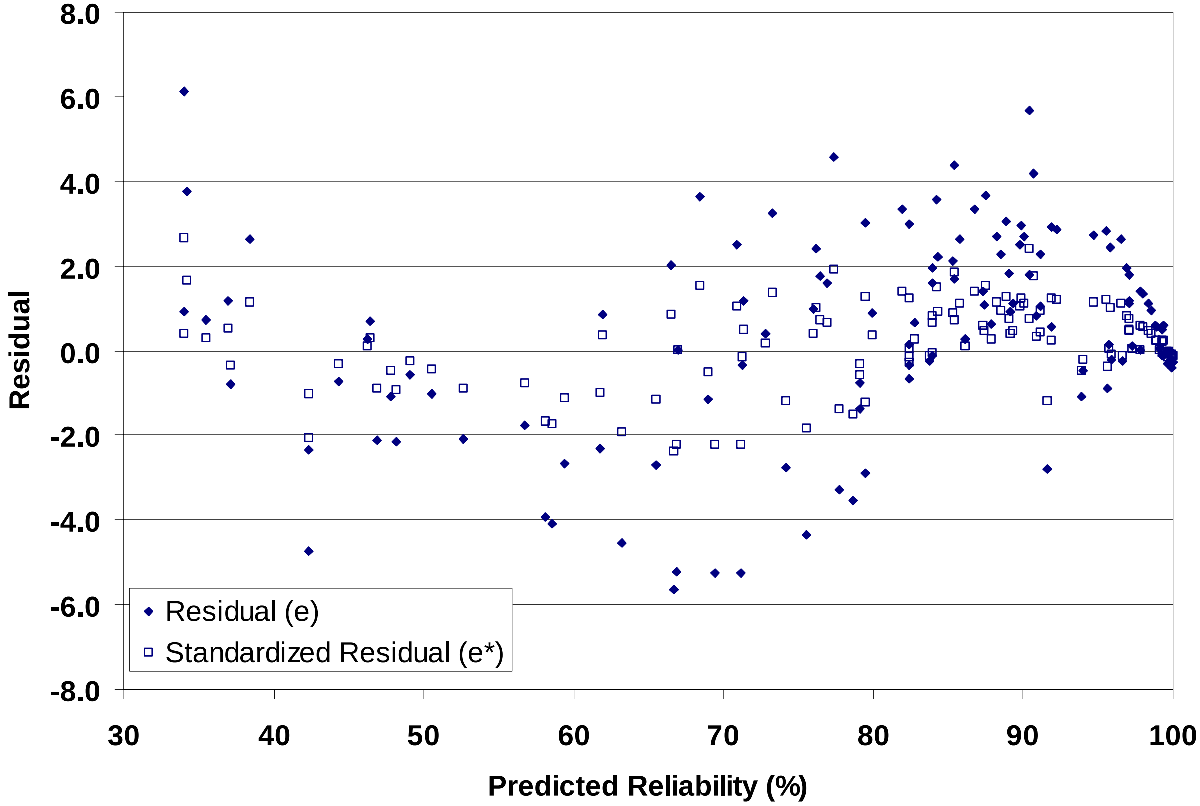

FIGURE 6 Residual Plot for Finn Fatigue (Medium CV)

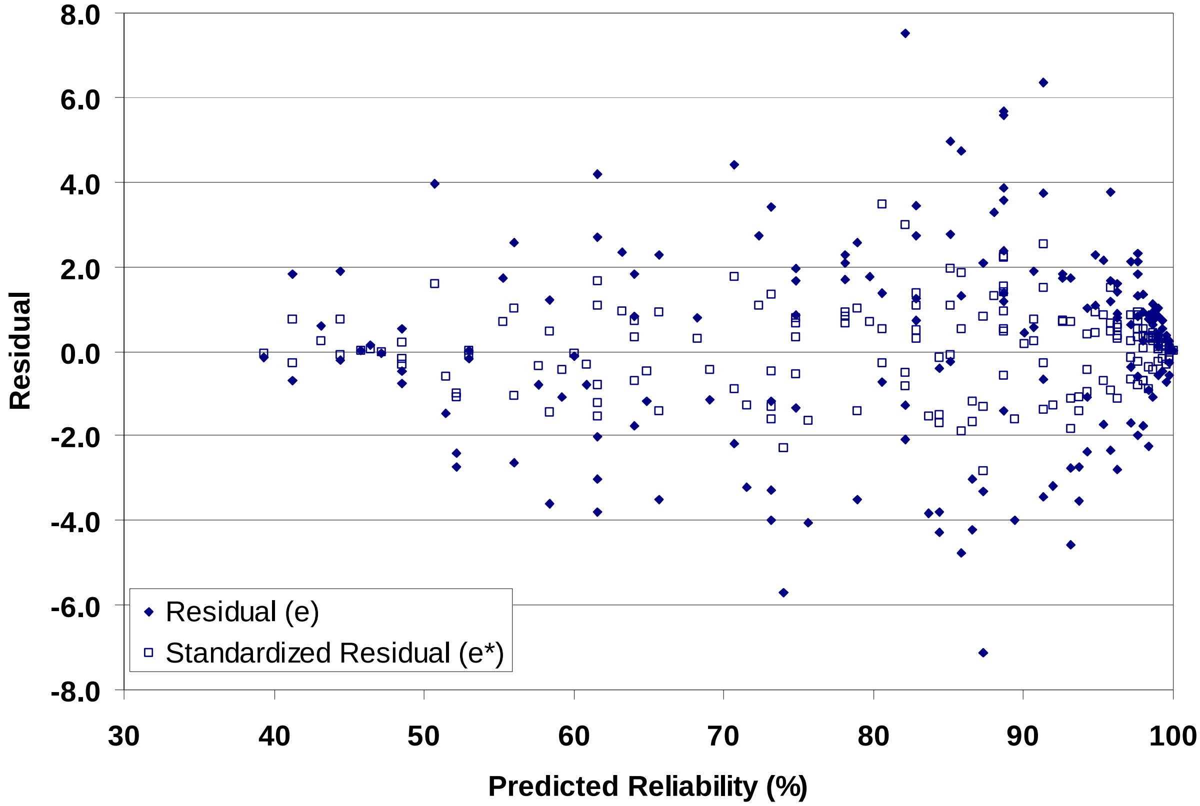

FIGURE 7 Residual Plot for Mn/ROAD Rutting (Low CV)