

Converting Between MnROAD Station/Offset and Wright County Coordinates (X,Y)

Bruce Tanquist

Equations for Linear Segments

$$\text{STATION} = X \cdot \cos\phi - Y \cdot \sin\phi + C1 \quad (1)$$

$$\text{OFFSET} = -X \cdot \sin\phi - Y \cdot \cos\phi + C2 \quad (2)$$

$$X = \text{STATION} \cdot \cos\phi - \text{OFFSET} \cdot \sin\phi + C3 \quad (3)$$

$$Y = -\text{STATION} \cdot \sin\phi - \text{OFFSET} \cdot \cos\phi + C4 \quad (4)$$

LVR North Segment

STATION 61+44.72 to 108+60.42

$$\phi = 0.65398941513$$

$$C1 = -299675.671 \text{ ft}$$

$$C2 = 493195.925 \text{ ft}$$

$$C3 = 537880.967 \text{ ft}$$

$$C4 = 209121.483 \text{ ft}$$

LVR South Segment

STATION 156+00.00 to 214+05.14

$$\phi = 0.65398941513$$

$$C1 = -289675.671 \text{ ft}$$

$$C2 = 493119.926 \text{ ft}$$

$$C3 = 529898.100 \text{ ft}$$

$$C4 = 215144.740 \text{ ft}$$

Mainline from end of Curve # 1019 to start of Curve #1020

STATION 1104+42.45 to 1105+83.56

$$\phi = 0.56672295253$$

$$C1 = -231494.077 \text{ ft}$$

$$C2 = 464906.152 \text{ ft}$$

$$C3 = 444897.678 \text{ ft}$$

$$C4 = 267942.707 \text{ ft}$$

Mainline from end of Curve # 1020 to start of Curve #1028

STATION 1121+57.96 to 1227+41.20

$$\phi = 0.65398941513$$

$$C1 = -189675.670 \text{ ft}$$

$$C2 = 493019.926 \text{ ft}$$

$$C3 = 450470.950 \text{ ft}$$

$$C4 = 275901.123 \text{ ft}$$

Equations for Curved Segments

$$STATION = PC \pm R \cdot (\theta' - \theta) \text{ ft} \quad (5)$$

$$OFFSET = \pm \left(\sqrt{(Y - Y_C)^2 + (X - X_C)^2} - R \right) \text{ ft} \quad (6)$$

$$X = X_C + (R \pm OFFSET) \cdot \cos \left(\theta \pm \frac{STATION - PC}{R} \right) \text{ ft} \quad (7)$$

$$Y = Y_C + (R \pm OFFSET) \cdot \sin \left(\theta \pm \frac{STATION - PC}{R} \right) \text{ ft} \quad (8)$$

Where:

STATION = Station (ft) of the unknown point

OFFSET = Offset (ft) of the unknown point

PC = Station (ft) of the point of curvature

(X, Y) = Wright County coordinates (ft) of the unknown point

(X_C, Y_C) = Wright County coordinates (ft) of the curve's center point

(X_{PC}, Y_{PC}) = Wright County coordinates (ft) of the point of curvature

R = Radius of the curve (ft)

$\pi = 3.141592654$

$$\theta = \tan^{-1} \left(\frac{Y_{PC} - Y_C}{X_{PC} - X_C} \right) \text{ (add } \pi \text{ for Curves 1019 and 1028)} \quad (9)$$

$$\theta' = \tan^{-1} \left(\frac{Y - Y_C}{X - X_C} \right) \text{ (add } \pi \text{ for Curves 1019 and 1028)} \quad (10)$$

Note: Differences in sign (\pm) and \tan^{-1} calculations are due the fact that in the coordinate system centered on (X_C, Y_C), Curve 1020 is in quadrant I while Curves 1019 and 1028 are in quadrant III. For quadrant I coordinates, the sign is (-). For quadrant III coordinates, the sign is (+) and π is added to θ and θ' .

Mainline Curve # 1019

STATION 1088+63.5 to 1104+42.45

Change \pm to +, add π to θ'

PC = 108863.50 ft

X_C = 547787.902 ft

Y_C = 223914.214 ft

R = 18093.404 ft

$\theta = 4.058399537$

Mainline Curve # 1028

STATION 1227+41.20 to 1261+52.87

Change \pm to +, add π to θ'

PC = 122741.20 ft

X_C = 561828.641 ft

Y_C = 219420.014 ft

R = 22918.312 ft

$\theta = 4.058399585$

Mainline Curve # 1020

STATION 1105+83.56 to 1121+62.5

Change \pm to -

PC = 110583.56 ft

X_C = 528479.33 ft

Y_C = 193308.92 ft

R = 18093.404 ft

$\theta = 1.004073400$

NOTES

Data used to develop equations 1 through 4 was obtained from the original MnROAD plans.

Linear Segments

Beginning and ending azimuth values for the curves were used to determine the azimuths of the linear segments, and from these azimuths (converted to radians), the ϕ values were calculated using Equation 5.

$$\phi = \text{azimuth} - \pi/2 \quad (5)$$

The ϕ values were used to rotate the known STATION/OFFSET coordinates into alignment with the X/Y coordinates and vice versa. The C values were calculated by substituting known station/offset and X/Y values into equations 1 through 4. C values were calculated at the beginning and ending of each line segment. Each pair was then averaged to obtain the final C values. Since the OFFSET values were 0 for all of the alignment data, known sensor locations were used to determine the sign of C2.

Curved Segments

There are three circular curves on the mainline portion of MnROAD. There are also curves in the East and West loops of the low volume portion, but since they contain no test sections, no conversions were done.

This conversion involved translation of the X/Y coordinates into a coordinate system with the curve's center point at its origin and then to polar coordinates. To determine stations, the point of curvature (PC) was used as a starting point. The station of an unknown point was obtained by adding the arc length to the PC. Offsets were accounted for by adding or subtracting them from the radius.

The equations for Curve 1020 have a slightly different form than those for Curves 1019 and 1028 because the translated coordinates are in different quadrants (Curve 1020 is in quadrant I and Curves 1019 and 1028 are in quadrant III). The value π was added to \tan^{-1} calculations for coordinates in quadrant III. There were also sign differences in some of the equations because while positive and negative offsets correspond to shorter and longer radii in quadrant I, they have the opposite effect in quadrant III.